Macroeconomic Theory
Comprehensive Exam
Answer All Questions
Point Totals Suggest Minutes Per Question

1 First Semester

1.1 Alternative Tax Policies (30 Points)

Households have time-separable utility over consumption and leisure with a period-utility function of the form \( u(c) + \nu(\ell) \), where both \( u \) and \( \nu \) are smooth, strictly increasing and strictly concave functions. The discount factor is \( \beta \). The total time endowment is normalized to one for each household. The production technology is given by \( F(k, n) \) and exhibits constant returns to scale.

1. Define a Tax-Distorted Competitive Equilibrium in the Neo-Classical growth model with taxes on labor income (tax rate \( \tau_{n,t} \)), gross capital income net of depreciation (i.e., \((r_t + 1 - \delta)k_t\) is taxed at rate \( \tau_{k,t} \)), and consumption expenditures (taxed at rate \( \tau_{c,t} \)). Prove that there is a unique non-trivial steady state if all tax rates are constant in \((0,1)\).

2. Consider a steady state in which the consumption tax rate is zero and the capital tax rate is positive. Show that there is an alternative tax system in which the capital tax rate is zero but there are positive consumption expenditure taxes. What must be true about the sequence of tax rates \((\tau_{c,t})_{t=0}^{\infty}\) in order for the two economies to have the same allocations?

3. Consider the problem of a Ramsey planner choosing optimal taxes rates. Is it true that consumption taxes must converge to zero in order for the TDCE to reach the optimum? Explain.
1.2 OLG in With Different Trading Opportunities (60 Points)

Consider an small open economy with an overlapping generations structure. That is, every period $t$ there are two types of households in the economy, those who have just been born and those that were born in $t-1$. Endowments are given by $e_y$ when a household is young and $e_o$ when they are old. Households value consumption only in the two periods in which they live according to:

$$U^t = \frac{1}{1-\sigma} [(c^t_y)^{1-\sigma} + (c^t_{t+1})^{1-\sigma}]$$

(a) Suppose the economy is closed, i.e. the only trade that can occur is between households of consecutive generations. Define and fully characterize a competitive equilibrium under the assumption of no free disposal.

(b) Now suppose that for $t \geq 0$ the households can borrow or save from abroad at the *exogenous* world interest rate $1 + r$. The initial old (those born at $t = -1$) have no debt or savings. Change the definition of equilibrium (i.e. add or remove one equation) to allow for this borrowing and saving from abroad and fully characterize the equilibrium as a function of $r$.

(c) True or false (and explain why): an increase in the world interest rate will lead to a fall in the amount of debt held by this country.

(d) Suppose now that there are two countries that can trade with one another. They have the same number of young and old households in each period. Country one has endowments $e^1_y = 1 + \epsilon$ and $e^1_o = 1 - \epsilon$. Country two has endowments $e^2_y = 1 - \epsilon$ and $e^2_o = 1 + \epsilon$. Define a world-wide equilibrium when the countries can borrow and lend between one another (but not from outside of these two). What is the market clearing interest rate? Hint - guess and verify after taking note of the symmetry!

(e) Now suppose that country one has fiat money. The initial old households in country one own the entire stock, given by $M = \epsilon$, at $t = 0$ and there are no money injections over time. Is there a monetary equilibrium in which country one has valued fiat money? If so then what is true about global welfare compared to the equilibrium in part (d)?
2 Second Semester

2.1 Optimal monetary policy with and without commitment (45 points)

The central bank’s objective is to minimize the loss function

\[ \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left[ \phi_p (\pi_{t+j})^2 + \phi_w (\pi^w_{t+j})^2 + \phi_x (x_{t+j})^2 \right] \]

subject to

\[ \pi_t = \beta E_t [\pi_{t+1}] + \kappa_p x_t + \lambda_p w_t \]
\[ \pi^w_t = \beta E_t [\pi^w_{t+1}] + \kappa_w x_t - \lambda_w w_t \]
\[ w_t = w_{t-1} + \pi^w_t - \pi_t - \hat{A}_t \]

where \( E_t \) is the conditional expectation operator, \( x_t \) is the central bank’s instrument, \( \pi_t, \pi^w_t \), and \( w_t \) are other endogenous model variables, and \( \beta, \phi_p, \phi_w, \phi_x, \kappa_p, \kappa_w, \lambda_p, \) and \( \lambda_w \) are (composite) model parameters. The central bank takes actions after the shock \( \hat{A}_t \) is realized. \( \hat{A}_t \) is iid over time and has a unit variance.

a) First, suppose that the central bank can credibly commit at date \( t \) to a contingent path for \( x_{t+j} \). Characterize, as far as you can, the solution to the optimal monetary policy problem above with commitment.

b) Next, suppose that the central bank cannot credibly commit and, instead, chooses \( x_t \) at each date. Characterize, as far as you can, the solution to the optimal monetary policy problem above without commitment.

c) Finally, consider the following relationships and restrictions between the model parameters

\[ \kappa_p = \lambda_p \left( \frac{\alpha}{1 - \alpha} \right); \quad \kappa_w = \lambda_w \left( 1 + \frac{\chi}{1 - \alpha} \right) \]
\[ \phi_p = \frac{\varepsilon_p}{\lambda_p}; \quad \phi_w = \frac{(1 - \alpha) \varepsilon_w}{\lambda_w}; \quad \phi_x = \left( 1 + \frac{\chi + \alpha}{1 - \alpha} \right) \]

\[ \kappa_p = \kappa_w; \quad \varepsilon_p = (1 - \alpha) \varepsilon_w \]

where \( \varepsilon_p, \varepsilon_w, \alpha \), and \( \chi \) are (deep) model parameters. Characterize fully the solution of the model under optimal policy given these relationships and restrictions. Is there a difference in outcomes when the central bank can and cannot credibly commit?
2.2 Monetary model with price-adjustment cost (45 points)

A general equilibrium monetary model where firms face price-adjustment costs.

**Consumer**

The representative consumer’s period utility is given by

\[ u(C_t, N_t) = \log C_t - \frac{N_t^{1+\phi}}{1+\phi} \]

where \( C_t \) is consumption of the composite final good, \( N_t \) is hours supplied, and \( \phi \) is the inverse of the Frisch elasticity of labor supply. The consumer discounts the future by the discount factor \( \beta \) and maximizes expected discounted utility over the infinite horizon.

The consumer can save in one-period risk-less nominal bonds that are in zero net supply. Assume that the consumer is subject to a no-Ponzi game constraint. The representative consumer owns all the firms in the economy and takes profit flow from them as given. The labor market is competitive.

**Firms**

There are two types of firms in the economy: composite final good producers and differentiated varieties producers.

**Composite final good producers**

This sector is perfectly competitive in which final good producers combine a continuum of differentiated varieties, indexed by \( i \), using a Constant Elasticity of Substitution technology

\[ Y_t = \left[ \int_0^1 y_t(i)^{\frac{1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \]

where \( Y_t \) is aggregate output of the composite final good, \( y_t(i) \) is output of variety \( i \), and \( \varepsilon > 1 \) is the elasticity of substitution among the varieties.

**Differentiated varieties producers**

This sector is monopolistically competitive and firms face price adjustment costs. A continuum of firms, indexed by \( i \), produce the differentiated varieties. Firm \( i \) produces variety \( i \) with a linear production function that uses labor as input

\[ y_t(i) = A_t n_t(i) \]

where \( y_t(i) \) is the output of firm \( i \), \( n_t(i) \) its labor input, and \( A_t \) an aggregate TFP shock that follows an exogenous stationary AR(1) process. Firms hire labor from a common, competitive market.

Changing nominal prices of differentiated varieties is costly and all firms face a price-adjustment cost \( q(.) \) given by

\[ q \left( \frac{p_t(i)}{p_{t-1}(i)} \right) \]

where \( p_t(i) \) is the nominal price of variety \( i \) and \( q(.) \) is a convex function that satisfies \( q(1) = q'(1) = 0 \).

Firms face a downward sloping demand curve for their varieties and maximize expected discounted profits over the infinite horizon. Firms also receive a constant subsidy on their total sales (total revenue) given by \( \tau \).
Government

There is no government spending in this model and we abstract from the financing of the lump-sum constant sales subsidy above. The central bank conducts monetary policy using a feedback interest rate rule that satisfies the Taylor Principle: it adjusts the nominal interest rate in response to (aggregate) inflation by enough.

a) Characterize the **first-best** (efficient) equilibrium in this model.

b) Characterize the **flexible-price** equilibrium in this model.

c) Consider the non-stochastic steady-states of the equilibria in a) and b). What value of $\tau$ is needed to ensure that the steady-states of a) and b) coincide?

From now on, consider this **common** steady-state above in c).

d) Characterize, as far as you can, the **log-linear sticky-price** equilibrium in this model. The log-linear approximation of the non-linear equilibrium conditions is around the non-stochastic steady-state from c) which features zero net inflation.