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THE UTILITY OF BEDROCK INCISION MODELS
How river channels encode information about active tectonics, and how and when we can decode it.

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THE UTILITY OF BEDROCK INCISION MODELS

How river channels encode information about active tectonics, and how and when we can decode it.

In all but the most arid of areas, networks of river channels develop in response to the flow of water from high to low elevations. These networks traverse environments with vast differences in climate, lithology, and tectonic setting (Fig. 1). Yet independent of these environmental differences, they all reach a characteristic form known as the graded river profile when allowed to equilibrate over time (Fig. 2). Modeling of the graded river profile has been motivated by the recognition that “if we can adequately describe the ideal, graded forms for particular rivers, then deviations from these forms can be considered as significant sources of information.” (Snow and Slingerland, 1987) In particular, these graded profiles could potentially reveal important information about the processes that shape them and their natural settings.

Empirical studies (Flint, 1973; Howard and Kerby, 1983) have found that these profiles can be modeled by a power-law relationship between slope ($S$) and upstream drainage area ($A$):

$$S = k_s A^{-\theta}$$  \hspace{1cm} (1)

The scaling of the slope-area relationship, as manifest in $k_s$ and $\theta$, varies considerably between environments, and depends on factors such as climate, bedrock type, and tectonic forcing. However, the consistency of the graded profile provides a framework for comparing these influencing factors between different channels, and even between portions of the same channel.
Figure 1. A photographic sampler of Earth’s diverse environments. Top: Hyperarid environments, such as Antarctica (left, by Chantal Steyn) and the Sahara (right, by VivaNOLA) are too dry for any significant water transport and sculpting to occur across their surface. Middle: Tectonically inactive environments such as the Amazon Basin (left, by Richard Hoff) and the Mississippi (right, by Katy Silberger) have limited topography and broad, meandering rivers. Bottom: Tectonically active regions like Iceland and the Himalaya (left/right, by author) are steeper and more dramatic, with rivers carving relatively straight courses and actively incising into bedrock. (Images used under Creative Commons licenses or with explicit authorial permission.)
Thus the deviations of rivers from the graded profile (Fig. 3) can, in principle, be used to pinpoint areas of change in the factors influencing $k_s$ and $\theta$.

Various attempts have been made to model the effects of bedrock, tectonics, and climate on channel profiles (e.g. Tucker and Slingerland, 1997; Whipple et al., 1999; Sklar and Dietrich, 2001; Duvall et al., 2003; Wobus et al., 2006). These efforts have been by far the most successful for tectonics (e.g. Wobus et al., 2006); bedrock type and climate are multifaceted phenomena and are less well suited to being quantified than rates of tectonic deformation (Sklar and Dietrich, 2001; Whipple, 2004 and sources therein). The principal utility of bedrock channel morphology as an investigative tool is therefore in constraining spatial variation in tectonics.

Figure 2. Longitudinal profile of the Calpfasture River, Va., showing the usual concavity of a graded river profile. (Hack, 1957)

The regression parameters $k_s$ and $\theta$ can be easily calculated from a digital topographic dataset (Wobus et al., 2006). The complexity of this method therefore lies in the interpretation of those data, as they contain convolved information about the various physical processes and properties that influence bedrock incision. Even if study areas are selected to minimize variation
in climate and bedrock lithology, few if any natural environments allow for those variables to be controlled completely; thus it is crucial to understand when and why the model is applicable. In order to illustrate the potential of this method and its limitations, I here review the scientific literature on (1) the origins of this model, which draws on both empirical data and their interpretation through the physical laws that govern bedrock incision; (2) the factors that influence the model fit, and how they affect the application of this model to complex datasets.

**Figure 3.** A theoretical longitudinal profile where a major knickpoint disrupts the grading of the river profile. Long profile data (elevation versus distance) are shown as solid lines with linear axes labeled on top and at right; slope-area data (log S vs. log A) as crosses with logarithmic axes labeled at bottom and at left. (Wobus et al, 2006.)

**Model Origins**

The characteristic upward concavity of river profiles has been recognized at least since the late 19th century. G.K. Gilbert provides the following description in his *Report on the Geology of the Henry Mountains* (1877, p. 110; see also Fig. 4),

> If we follow a stream from its mouth upward and pass successively the mouths of its tributaries, we find its volume gradually less and less and its grade steeper and steeper, until finally at its head we reach the steepest grade of all. If we draw the
profile of the river on paper, we produce a curve concave upward and with the
greatest curvature at the upper end. The same law applies to every tributary ...  
The nearer the water-shed or divide the steeper the slope; the farther away the less 
the slope.

Figure 4. G.K. Gilbert’s 1877 sketch of the characteristic slopes of river profiles.

Though this general shape shows great consistency across a diversity of environments, it
is critical to note that it arises as a result of gradual equilibration over time, as the river carves for
itself an energetically favorable path from its headwaters to base level. Striking examples of this
gradual adjustment may be seen in the young volcanic island of Iceland. Figure 5 shows the
profiles of two Icelandic rivers, a) Þjórsá, a young river that traverses Iceland’s active volcanic
zone, and b) Hvítá, an equilibrated river from an older, inactive part of the island. The youthfully
rugged profile of Þjórsá bears little resemblance to Gilbert’s upward-concave ideal at present, but
with time, rivers such as this tend to carve their way back to a smoother, concave-up profile.
Since few fluvial networks are young enough to be in such dramatic disequilibrium, Gilbert’s
postulation holds true across most natural environments.

Figure 5. Comparison of a non-graded disequilibrium profile with a graded profile. Left: The long profile of Þjórsá, 
Iceland, a young and rugged river that traverses Iceland’s active volcanic zone. Right: The long profile of Hvítá, 
Iceland, an equilibrated river from an older, inactive part of the island. (Proximal source: geology lecture notes from 
Auður Ingimarsdóttir, Hamrahlið Secondary School, Iceland.)
In the century that has passed since Gilbert’s pioneering work, various efforts have been made to model this characteristic shape and relate its development to the physical process of erosion. Numerical modeling of channel elevation as a function of distance suggests that exponential, logarithmic, and power-law curves can all fit stream profiles closely (Snow and Slingerland, 1987; Fig. 2 shows an example of a logarithmic fit). A critical breakthrough was made in the 1970s with the recognition that channel slope can be expressed as a power-law function of drainage area (Flint, 1973; Eq. 1). An example of this may be seen in Figure 3, where a river channel with a prominent knickpoint has been plotted both in physical space and in slope-area space. The anomalously high slope of the knickpoint causes it to rise above the overall linear expression of the channel in slope-area space.

Spatial and temporal variations in climate, tectonics, and bedrock lithology exert a substantial influence on the scale over which the graded river profile is stretched and on local deviations from its ideal form. The ultimate aspiration of bedrock incision modeling is to reconstruct those variations from the effect they have on river profiles. Such a reconstruction depends on the existence of models relating quantifiable properties of climate, tectonics, and lithology to bedrock channel form.

Of these three factors, by far the best constrained is tectonic influence. This is not least because tectonics are more readily quantifiable than either climate or lithology, and thus better suited to mathematical modeling. All three phenomena certainly have quantifiable aspects such as rainfall intensity, rock tensile strength, or the slip rate along a fault. In the case of climate and lithology, however, the sheer number of quantities needed to represent them in adequate detail for this model appears to exceed human powers of observation, at least for the present. Tectonics,
on the other hand, acts on bedrock channels primarily through vertical motion of rock, i.e. tectonic uplift. This is a single numerical quantity that varies in space and time, and its influence on channel form is therefore comparatively simple to understand.

The point of departure for modeling tectonic influences on channel incision is the so-called “channel profile evolution equation”, which in its most general form is a simple statement of conservation of bedrock mass for an eroding channel. The change in surface elevation, $dz/dt$, is determined by the balance between uplift, $U$, and erosion, $\varepsilon$:

$$dz/dt = U(x,t) - \varepsilon(x,t)$$  \hspace{1cm} (2)

This formulation serves as a framework for investigating the coupling of tectonics and surface processes.

If two of the terms in the channel profile evolution equation are observed or controlled for, the behavior of the third as a function of other influencing factors can be observed. This fact was cleverly utilized by Howard and Kerby (1983). Very rarely do geologists have a chance to perform something analogous to a controlled experiment, but in 1963 an abandoned borrow pit near Stafford, Virginia, presented just such an opportunity. In the authors’ own words,

Occasionally, nature presents us with a microcosm, scaled-down temporally and/or spatially while maintaining many essential features of larger systems. Such appears to be the case in natural and man-induced badland landscapes, where the absence of vegetation and the soft rocks greatly accelerate landform development. ... Thus, badlands have been intensively studied both because of their morphometric similarity to large-scale landforms and because of the possibility of short-term measurements of process rates.
In addition to rapid changes in surface elevation (which imply large $dz/dt$), the Stafford borrow pit had a further advantage: The site is a tectonically stable region where $U$ has an uniform zero value. Therefore:

$$\frac{dz}{dt} = - \varepsilon(x,t)$$ (3)

This allowed for direct quantification of erosion rates through surveying of the topography, which the authors performed repeatedly over a 15-year period between 1963 and 1978. They found that erosion rate could be expressed as a power-law function of drainage area ($A$) and channel gradient ($S$),

$$\varepsilon = KA^mS^n$$ (4)

Howard and Kerby show that this behavior is consistent with a simple physical model that assumes erosion rate to be proportional to the stream power of the flow, i.e. the rate of energy dissipation against the channel bed. A complete derivation of this model (Whipple and Tucker, 1999; see Appendix) combines this assumption with conservation of water mass and momentum, as well as power-law relations for hydraulic geometry and basin hydrology. The resulting equation has the exact form shown in Equation 4, above.

Three new quantities, $K$, $m$, and $n$, arise from fitting real data to this general model. $K$ is known as the “coefficient of erosion” (Whipple and Tucker, 1999) or “erodibility”, and represents the ease or difficulty with which a given landscape can be eroded; the exponents $m$ and $n$ are less easily explained in terms of observable physical phenomena, but nonetheless play an important part in the model’s behavior, which will be discussed in more detail later.

A river channel can be divided into three reaches that are characterized by fundamentally different erosional processes. The colluvial channel at the river’s headwaters is dominated by
non-fluvial mass wasting; it is followed by the bedrock channel, where fluvial erosion of bedrock dominates and alluvial cover is patchy; last is the alluvial channel, which occurs below where the river transitions from erosion to sedimentation and derives its name from the thick alluvial cover produced by this sedimentation (Whipple, 2004). As is shown in Figure 6, all three channel reaches have a distinct expression in slope-area space. It is important to note that the empirical data used by Howard and Kerby in their analysis came only from bedrock channels, and the stream power erosion model deals only with such channels. However, as bedrock channels typically preserve 80-90% of drainage basin relief, their behavior determines the first-order response of topography to external influences (Whipple et al., 1999).

Figure 6. The division of an orogen into colluvial, bedrock, and alluvial channels, represented schematically in slope-area space. (Duvall et al., 2004)

A substitution of Howard and Kerby’s result (Eq. 4) into the channel profile evolution equation relates channel slope and area, properties that together define the graded river profile, to rates of tectonic activity:

\[
\frac{dz}{dt} = U(x,t) - K A^n S^a
\]  

\( (5) \)
This equation eliminates one unknown (erosion rate) from Equation 2 and bridges the gap between readily computed topographic metrics and tectonic rates – at least in theory. In practice, it is difficult to apply as such because it involves both surface uplift ($\frac{dz}{dt}$) and rock uplift ($U$).

A way around this difficulty is offered by the fact that $\frac{dz}{dt}$ is often so much smaller than $U$ that it can be treated as having zero value. This assumption, known as the steady-state assumption, derives its legitimacy from the fact that landscape evolution is driven by a negative feedback system that tends to minimize $\frac{dz}{dt}$ as time progresses (G. Hilley, pers.comm., 2009).

If uplift rates are much higher than erosion rates, the result will be an overall increase in channel slope due to positive $\frac{dz}{dt}$, which will cause erosion rates to increase, lowering $\frac{dz}{dt}$. The converse is true for situations where $\frac{dz}{dt}$ is negative, i.e. erosion rates are higher than uplift rates. Landscapes thus exhibit an overall tendency toward equilibrium if the tectonic event producing the uplift lasts much longer than the time it takes the fluvial system to adjust, as was argued by Hack (1960).

This critical assumption is not everywhere true; a study from the Washington Cascades examines a landscape that is in profound topographic disequilibrium (Moon et al., in review). However, in areas where this assumption is true and $U >> \frac{dz}{dt}$, the topography may be assumed to be in steady state. Equation 5 can then be simplified by setting $\frac{dz}{dt}$ to be zero:

$$U = K A^m S^n$$

Ignoring for the present the regression parameters $K$, $m$, and $n$, Equation 6 allows for direct calculation of uplift rate from $A$ and $S$, both of which can readily be computed from digital topographic data. Solving this equation for slope yields the expression
\[ S = \left( \frac{U}{K} \right)^{1/n} A^{-m/n} \]  

(7)

This expression has the exact same form as Equation 1, which describes the shape of a river profile as a function of slope and area. The equations’ parameters are related as follows:

\[ k_s = \left( \frac{U}{K} \right)^{1/n}; \quad \theta = \frac{m}{n} \]  

(8a, b)

\( k_s \) is referred to as the index of steepness and \( \theta \) as the concavity index, respectively. Neither of these is exactly equivalent to the geometric quantity it is named for; however, steepness in particular may be looked on as a slope value that has been normalized to its drainage area.

Wobus et al. (2006) describe a method for using this relationship to quantify tectonic activity. Taking the logarithm of Equation 1 yields

\[ \log S = \log k_s - \theta \log A \]  

(9)

This equation is linear in \( \log S - \log A \) space with a slope of \(-\theta\) and a \( \log S \)-intercept of \( \log k_s \), which in turn is proportional to uplift rate \( U \). The linearity of the expression allows for simple least-squares regressions to be performed on slope-area data. Because there are two unknown quantities in this equation, \( k_s \) and \( \theta \), there will necessarily be some covariation if both of them are allowed to behave as free parameters for a regression – a change in the slope of a line through a cluster of points can will affect the line’s intercept, and vice versa.

Fortunately, concavity values vary little throughout the world, falling in the range of 0.35-0.65 with a mean of about 0.5 (Wobus et al., 2006; Whipple and Tucker, 1999). Graphically, this translates into a relatively consistent slope of the log-slope/log-area regression, i.e. a similar rate of slope decrease with increasing drainage area. Furthermore, dimensional analysis has shown that concavity plays no direct role in the sensitivity of channel gradient and relief to the dimensionless group that governs the dynamics of the channel profile evolution.
equation (Whipple and Tucker, 1999). One variable can thus be eliminated from the analysis by choosing a fixed “reference concavity”, usually the mean concavity found from a free regression of slope and drainage area, and calculating all steepness values relative to this fixed concavity (Wobus et al., 2006). These normalized steepness values can then be compared with one another in a meaningful way. Since the concavity index is independent of uplift rate, this does not impact the method’s ability to capture variations in uplift rate.

Computing steepness relative to a reference concavity has another advantage: it allows a separate steepness value to be calculated for each location along a bedrock channel, effectively normalizing slope to the drainage area at which it occurs. In a perfectly graded channel, such as the one in Figure 2, every single point should fall on the same line in slope-area space and have the same $k_s$ relative to the reference concavity, which is also the slope of the line. In a more complex channel whose shape deviates from this simple form, normalized steepness values may vary along the length of the channel. An example of this may be seen in Figure 7, which shows a sharp spike in the steepness of several streams where they cross the San Gregorio Fault in California. In settings where steepness can be interpreted as reflecting tectonic uplift rates, maps of these along-channel variations in steepness can be treated as proxies for relative uplift rate. But what are the conditions under which steepness and uplift rate are so closely related? What factors can distort or disrupt that relationship, and how are these interplays expressed in the bedrock incision model proposed by Howard and Kerby and developed by Whipple, Tucker, Wobus, and colleagues? And does the natural world, in its infinite complexity, even offer any settings where the model’s prerequisite conditions are satisfied?
Figure 7. Steepness values from along the San Gregorio Fault, showing a large increase in steepness where drainage channels cross the fault (red line). This increase is most likely caused by a local spike in uplift rate due to pressure ridge formation along the eastern side of the fault. Cold colors are low steepness, hot colors high steepness. (Guðmundsdóttir and Hilley, 2009)

**INTERPRETIVE CHALLENGES**

The above discussion has proceeded for the most part without mention of the three parameters of Howard and Kerby’s original bedrock incision model, K, m, and n. However, the value of these quantities is of paramount importance if steepness is to be interpreted reliably as a proxy for uplift rate. As mentioned above, Whipple and Tucker (1999) demonstrated that the concavity
index, i.e. the ratio of m to n, plays no direct role in the dynamics of the channel profile evolution equation. On the other hand, the value of the “slope exponent” n by itself is critically important. This may be seen directly from Equation 8a, which shows that steepness is proportional to the nth root of uplift rate. Empirically determined values for n vary considerably from study to study (e.g. Wobus et al, 2006; Howard and Kerby, 1983), ranging from about 0.5 to as high as 2, though recent studies seem to be converging on a value around or a bit above 1 (G. Hilley, pers.comm., 2009). In any case, the uncertainty surrounding the exponent n is one of several barriers that prevents reliable conversion of steepness data into absolute uplift rates. Yet n does not seem to be easily linked to physical quantities that could be independently constrained, and so remains elusive even under the best of circumstances.

Even more important than n is K, commonly referred to as “erosivity” or the “coefficient of erosion” (Whipple et al, 1999). As seen in Equations 7 and 8a, erosivity and uplift rate have opposite effects on steepness, with an increase in one eliciting the same change in steepness value as a decrease in the other. From a physical standpoint, erosivity represents the ease or difficulty with which erosion can act in a given area. As such, it incorporates the effects of climate, which controls the supply of water in a fluvial system; lithology, which determines the degree to which the channel bed resists the action of the erosive agent; and other quantities such as sediment flux and transport capacity (Whipple and Tucker, 1999). Strong variation in any or all of these phenomena could dominate observed patterns of channel steepness and mask the effect of changes in uplift rate. Before steepness is interpreted as an uplift proxy, it is therefore necessary to assess the possible interference of both climate and lithology.
Steepness, and the factors that influence it, can vary either in space or in time. Spatial variation in uplift rates can be either gradual, as with a fold or a tilting block, or abrupt, as across a fault; uplift rates can also vary in time if tectonic conditions change. Likewise, the interference by $K$ can be either spatial or temporal in nature. Lithology, while largely temporally constant over geomorphic timescales, can have substantial spatial variation. Climate, on the other hand, can vary greatly in time as well as in space. Since the topographic data that form the basis for these analyses are essentially constant in time, they capture only the spatial manifestation of both permanent and transient signals.

Figure 8. Schematic long profile and map view plots comparing transient and steady-state systems. (A) Transient long profile showing short oversteepened reach separating old and new equilibrium states. (B) Profile crossing from one uplift regime to another, showing channel reaches with constant $k_s$ values separated by a high or low concavity transition zone in between. $A_c$ marks transition to fluvial scaling (see text); slope of log $S$ vs. log $A$ scaling is the concavity index $\theta$; y-intercept is the steepness index $k_s$. (C) Transient wave of incision propagates through system at a nearly constant vertical rate; knickpoints (white dots) should therefore closely follow lines of constant elevation (dashed line) in plan view. (D) In contrast, knickpoints separating zones of high and low uplift rates (white dots) follow the trend of the accommodating shear zone in map view (dashed line). In many cases, the map pattern of knickpoints may be a better diagnostic of a transient state than long profile form. (Wobus et al., 2006.)
Landslces with significant transient perturbations should theoretically not be examined using this method, because they violate the steady-state assumption. However, transient and steady perturbations to channel profiles can often be distinguished by their very different signatures both in map view. Figure 8 (Wobus et al., 2006) provides a comparison of a set of transient knickpoints migrating up through a drainage network (Panels A and C) and a set of permanent knickpoints fixed to a fault (Panels B and D). In the former case, the knickpoints migrate up the drainage network at a near-constant vertical rate and are consequently found at similar elevations. In the latter, the knickpoints are spatially aligned because they are fixed to the discontinuity in uplift rate across the fault. Such a pattern could not be generated by a transient signal migrating upstream. Wobus et al. (2006) therefore advocate the use of map-view examination for differentiating permanent and transient signals.

The knickpoints in Panels B and D mark the junction between two reaches of the same channel that have equilibrated to different values of U/K and consequently have different steepness values. This could also be the case for transient knickpoints if they represent a transition between different U/K regimes (Fig. 9). Thus the slope-area expression of a channel discontinuity alone cannot be used to determine whether it is transient or permanent. During a transition between two steady states, as predicted by Whipple et al. (1999) in Figure 9, the lower reaches of the channel have a steady form that is adjusted to the new U/K value, whereas the unadjusted headwaters of the channel still maintain their steady-state form from the previous U/K regime. These two are separated by a relatively brief transition zone where the channel is in true disequilibrium. While the steepness of the headwaters is not relevant to current values of U/K, it does constitute a paleorecord of past conditions. In conjunction with careful application of
Wobus et al.’s map view criterion, this fact could be used to characterize and even quantify the magnitude of abrupt changes in $U/K$.

![Figure 9](image.png)

**Figure 9.** Numerical simulation of the transient response of a channel profile to a sudden, uniform increase in erosivity. ... Inset shows the transient response in slope-area space (log-log scale). Both the main panel and inset shows the initial steady-state profile (T0; solid grey), three intermediate transient profiles, sequentially T1, T2, and T3 (black dashed lines), and the final steady-state profile (solid black). The transient response is characterized by an upstream-propagating wave of gradient, and therefore relief, reduction. ... Note that if $u$ were unchanged over the first kilometre of channel length and doubled only downstream of this point, the final steady-state solution would approximately coincide with the T2 transient profile. (Whipple et al., 1999.)

Climate exerts a variety of important influences on the coefficient of erosion $K$, the most obvious ones being related to the amount of precipitation. Since climate dictates the availability of water, it is directly related to the power that the fluvial system can bring to bear in eroding the channel bed. An increase in precipitation, all other things being equal, should therefore result in an increase in $K$, since $K$ is proportional to erosion rate (Eq. 4). An example of this may be seen in Figure 10 (Moon et al., in review), which depicts a strong inverse correlation between mean annual precipitation and cosmogenic radionuclide erosion rates across the Washington Cascades. Whipple et al. (1999) model the effects of a humidification of climate, represented by a doubling in $K$, on channel steepness and topographic relief, and find that it results in an overall decrease of both (Fig. 9). This flattening of the landscape is in accordance with the inverse proportionality of $k_s$ and $K$ (Eq. 8a).
Figure 10. Mean annual precipitation (left) and cosmogenic radionuclide erosion rates (right) from the Washington Cascades show a clear positive correlation between precipitation and erosion, and thus between precipitation and $K$. Note the reversed color bars: high erosion rates are red, high MAP is blue. (Moon et al., in review.)

But while general modeling of landscape response to climate change is possible, it is far more difficult to convert an observed climatic paradigm into a map of predicted $K$ values. Complexities such as the intensity and duration of rainfall events, the seasonality of precipitation, and the effects of freeze-thaw weathering result in highly complex behavior that cannot be adequately quantified by any known method. Significant spatial or temporal variation in any of these can produce a strong signature that masks topographic expression of uplift rate, and because of the difficulty of modeling their effects on erosivity, it is impossible to subtract out that signature. The most practical alternative remains to select study areas where climatic variation is minimal in order to avoid the issue entirely. Since many environments have relatively consistent macroclimatic patterns, this is a significant but not an insurmountable obstacle.

Much like climate, bedrock lithology has a strong yet complex impact on erodibility. It is well known that the more competent a given lithologic unit, the less readily it can be eroded, and the higher its value of $K$; in cases of abrupt spatial contrasts in rock competence, this can produce large knickpoints and spectacular waterfalls. A classic example of this is Niagara Falls,
where a hard limestone caprock over a soft shaly formation has created a prominent break in the river profile (Encyclopædia Britannica). This general expectation is also borne out on a regional scale across more gradual changes in lithology, as is the case in California’s Santa Cruz Mountains (Guðmundsdóttir and Hilley, 2009). The mean steepness of lithologic units in the Santa Cruz Mountains varies considerably between rock types, and correlates systematically with the expected mechanical properties of the units (Fig. 11).

Some effort has been made to quantify the effect of different lithologies on erosivity. Laboratory studies using abrasion mills have found that erosion rates scale inversely with the square of rock tensile strength (Sklar and Dietrich, 2001; Fig. 12). However, the difference between the mechanical properties of a single unfractured cylinder of rock and those of that same
rock with a field-scale distribution of fractures and heterogeneities is substantial enough to call
the continuum assumption into question (Pollard and Fletcher, 2005, p. 124–127; Whipple,
2004). Without careful consideration of the field-scale properties of each lithologic unit, Sklar
and Dietrich’s result cannot be used to predict variation in erosivity based on geologic map units.

![Variation in measured erosion rate in an abrasion mill with rock tensile strength. Each data point represents mean of 12 to 20 measurements of tensile strength and two to six replicate erosion runs; error bars are omitted because standard errors of these means are smaller than data symbols. (Sklar and Dietrich, 2001)](image)

Given all these constraints and confounding factors, are there any natural environments
where channel steepness truly reflects rates of tectonic deformation? A case study from Nepal
demonstrates that such places can indeed be found, especially if variables like climate and
lithology are carefully controlled for (Wobus et al., 2006). The uniform lithology and climate of
the Siwalik Hills in Nepal make it an ideal environment for testing this model; furthermore, the
uplift rate distribution of the area is well documented from a series of preserved and dated
bedrock terraces and fault-bend fold modeling (Hurtrez et al., 1999; Lavé and Avouac, 2000; Fig.
13B).
Figure 13. Comparison of uplift rate (here termed incision rate) estimates derived from (A) bedrock incision model, assuming a stream-power incision rule with $n = 1$, and (B) structural study of Hurtrez et al. (1999) and Lavé and Avouac (2000). Note general correspondence of location and magnitude of high incision rate zones from the two methods, suggesting that quantitative estimates of incision rates may be attainable in regions with relatively simple patterns of lithology and uplift. (Wobus et al., 2006) Note that the figures have no scale bar in the original publication; my best estimate of the scale is that the height of the figures is approximately 100-150 km.

Figure 14. Plot of normalized steepness index ($k_{sn}$) versus uplift rate ($U$) for seven strike-parallel channels from the Siwalik Hills in southern Nepal. Data suggest a linear correlation between $k_{sn}$ and $U$ with a zero intercept, consistent with a stream power scaling with $n = 1$. Wobus et al., 2006)

Wobus et al. (2006) computed channel steepness across the range and compared the values they found with the preexisting uplift rate data (Fig. 14). They found a near-perfect linear correlation between steepness index and uplift rate. This not only validated the initial assumption of near-uniform $K$, but also allowed the authors to ascribe specific numerical values to $K$ and $n$ using Equation 8a, $k_s = (U/K)^{1/n}$. The linear relationship indicates that $n$ is close to 1, and $1/K = 5.04$ yields a $K$ value of around 0.2. These numbers could then be used to convert a map of steepness values a map of uplift rates (Fig. 13A). A comparison of this result with the structural
uplift rate map in Figure 13B shows that the broad trend captured by the linear comparison (Fig. 14) is sustained at a comparable level of detail to that of the structural data, with the location and magnitude of high-uplift zones quite well represented. While this exceptional model fit was made possible by the availability of good control data, it might be possible to use a calibrated model such as this one in nearby areas where other approaches such as river terrace dating were not successful.

**Conclusions**

The physical principles relating bedrock channel morphology to uplift rate have been exactingly investigated and documented over the past forty years. In addition to pioneering empirical studies of channel morphology (e.g. Flint, 1973), theoretical and modeling efforts (Howard and Kerby, 1983; Whipple and Tucker, 1999; Wobus et al., 2006) have solidified our understanding of the bedrock incision model and its potential applications in mapping tectonic rates. Analysis of bedrock channel steepnes can offer a wealth of information about tectonics if performed in the right setting and with careful consideration of the assumptions implicit in the model. However, the application of this theory is made difficult by the variability exhibited by climate and lithology in most natural settings, as well as by the lack of independent constraints on the slope exponent n. At present, applications are thus restricted to areas where variability in climate and erosion is limited. Given the potential of this method, further investigation of these confounding factors seems a worthy enterprise. Yet in light of the complexity of the phenomena in question, it remains uncertain whether it will ever be possible to separate out their individual contributions to a given region’s topography.
**APPENDIX: DERIVATION OF THE STREAM POWER EROSION MODEL**

This derivation is based on the one presented in Whipple and Tucker (1999).

**Variables**

- **Q** discharge \([\text{L}^3 \text{T}^{-1}]\)
- **V** velocity \([\text{L} \text{T}^{-1}]\)
- **D** average flow depth \([\text{L}]\)
- **W** channel width \([\text{L}]\)
- **\(\tau_b\)** basal shear stress \([\text{M} \text{L}^{-1} \text{T}^{-2}]\)
- **\(\rho\)** density of water \([\text{M} \text{L}^{-3}]\)
- **S** gradient of channel bed
- **A** upstream drainage area \([\text{L}^2]\)

**Dimensional Constants**

- **\(k_b\)** total erodibility \([\text{M}^{-a} \text{L}^{3a-1}]\)
- **\(k_q\)** discharge-area coefficient (effective precipitation) \([\text{L}^{3-2c} \text{T}^{-1}]\)
- **\(k_w\)** channel width-discharge coefficient \([\text{L}^{1-3b} \text{T}^{b}]\)

**Fundamental Relations**

1. \(Q = VD W\)  
   conservation of mass
2. \(\tau_b = \rho g D S\)  
   cons. momentum, steady and uniform flow in wide channels
3. \(W = k_w Q^b\)  
   hydraulic geometry of the channel
4. \(Q = k_q A^c\)  
   basin hydrology
5. \(\varepsilon = k_b (\tau_b V)^a\)  
   erosion rate is proportional to stream power

**Derivation**

- \(\varepsilon = k_b (\rho g D S V)^a\)  
  substituting (2) into (5)
- \(\varepsilon = k_b (\rho g Q S / W)^a\)  
  from (1), \(DV = Q/W\)
- \(\varepsilon = k_b (\rho g Q S / (k_w Q^b))^a\)  
  substitute (3) for \(W\)
- \(\varepsilon = k_b (\rho g Q^{(1-b)} S k_w^{-1})^a\)  
  simplify
- \(\varepsilon = k_b (\rho g k_q^{(1-b)} A^{c(1-b)} S k_w^{-1})^a\)  
  substitute (4) for \(Q\)

Grouping the constants together, we find

\[
\varepsilon = k_b (\rho g k_q^{(1-b)} k_w^{-1})^a A^{ac(1-b)} S^a
\]

This equation has the same form as the empirical relation found by Howard and Kerby (1983):

\[
\varepsilon = K A^m S^n
\]

The constant \(K = k_b (\rho g k_q^{(1-b)} k_w^{-1})^a\), and the exponents \(m = ac(1-b)\) and \(n = a\), respectively.
REFERENCES


