§ 8. Logic as a theoretical discipline.

Philosophical reflection on the nature of logic is seldom to be found in the writings of modern analytic philosophers. What one finds instead are discussions of the application of individual logical systems, both within philosophy and in extra-philosophical disciplines such as mathematics and physics. One then finds more or less explicitly pragmatic defences of one or other system or logical regime as yielding the most beneficial results in such application. Where such reflection is engaged in, one finds too frequently that a conception is advanced of logic as a purely normative science. That is, it is held to be a sufficient determination of the nature of logic to characterise it as a discipline which tells us how we ought to think, reason, argue, construct theories and proofs, and so on. But norms which do not sustain justification are valueless. That is to say, any normative science which is to do work which is scientifically valid, presupposed one or more theoretical discipline as its foundation. The latter would tell us why we ought to submit to the norms in question.

The temptation, against the pragmatic background just mentioned, is to suppose that the results of the necessary theoretical discipline of logic could be obtained merely empirically. Experiments of the relevant type would show, in an ad hoc way, that given logical systems yield such and such beneficial results, therefore the norms on which these systems rest are to be accepted for such and such purposes. Such an approach however, would ignore the fact that it is precisely the notion of 'beneficial result' which is at stake; it would feed off the pragmatic
prejudice that the notion of 'benefit' is unproblematic.

As Husserl puts it, 'Any normative proposition of e.g. the form
"An A should be B" implies' - not, indeed, any kind of empirical proposition
but, rather, -

the theoretical proposition "Only an A which is B has the properties
Beschaffenheiten, determinations) C", in which 'C' serves to
indicate the constitutive content of the standard-setting predicate
'good' (e.g. pleasure, knowledge, whatever, in short, is marked down
as good by the valuation fundamental to our given sphere). The new
proposition is purely theoretical: it contains no trace of the
thought of normativity. (LU,48; LI, 87f).

What, then, is to be the subject-matter of this theoretical discipline
which is determined for the case of logic? Or ought we, perhaps, to establish
first of all that a 'subject-matter', a pre-existing domain of theoretical
investigation is truly needed by such a discipline? Husserl is fond of
the suggestion that the paradigm for each of the theoretical disciplines
which would stand prior to the three traditional normative disciplines of
philosophy - logic, ethics, and aesthetics - is provided by pure mathematics.
But he himself is ready to point out that the mathematician himself proceeds
merely as the

ingenious technician, the constructor, as it were, who, looking
merely to formal interconnections, builds up his theory like a
technical work of art. (LU,253; LI,244).

Thus the subject-matter of the special science of mathematics has arisen
piecemeal with the development of mathematical concepts, methods and skills,
and the mathematician would not have been aided in this progress if the
philosopher had been somehow able to delineate in advance the scope of
his investigations (nor, a fortiori, if the philosopher had taken it upon
himself to perform the tasks of the mathematician as such). The suggestion
is, now, that the theoretical science of logic could also proceed in this
manner, that is to say in the manner in which the actually existing mathem-
atical logic has developed, as a 'free science' (Becker, 1959, Ch.3) generat-
ing various technical calculi fashioned, stage by stage, with various logical
or extra-logical purposes in mind.

It is not immediately clear that what is acceptable - or is thought to be acceptable - in the case of mathematics can be automatically applied also in the case of logic. For the mathematician, as Russell points out, is not strictly speaking a theoretician at all:

As the practical mechanic constructs machines without needing to have ultimate insight into the essence of nature and its laws, so the mathematician constructs theories of numbers, quantities, syllogisms, manifolds, without ultimate insight into the essence of theory in general, and that of the concepts and laws which are its conditions. (LI, 244f.)

The like holds of all 'special sciences', as Russell states, but yet mathematics is admitted as a special science. Could not the same be said also for (contemporary, mathematical) logic? To feel our way here it is necessary to distinguish between three broad 'levels' of theoretical activity:

(i) the first level, the level of **ars inventiva**, (of the mechanic toying with the fault in his machine and thereby putting nature - in this small area - to the test), is 'theoretical' only in the weakest sense. Nevertheless there are certain kinds of 'invention' which are such that they evolve into activities (networks of acts) on the second level;

(ii) the level of **special sciences**, which are distinguished not by any peculiarity of subject matter but by the fact that they are 'positive' disciplines whose theoretical foundations are unsecured. What is truly valuable, truly scientific, amongst such disciplines is, as we shall see, capable of being correlated with a corresponding discipline on the third level:

(iii) the level of **rigorous science**, of **strange Wissenschaft**, which alone is such as to contain 'perfect, pure, genuine theoretical knowledge' (LI, 254; LI, 245).

The crux of our problem can now be expressed as follows: does (mathematical)
logic - conceived not as a sub-discipline of mathematics but as a discipline which would provide the theoretical foundation for the normative science of logic which is known to the tradition - have as secure a place within the second level as does 'standard' mathematics, and can either of these two disciplines be correlated with a strange Wissenschafter level (iii) ?

To answer these questions it is necessary to state much more precisely the criteria which must be met before passage (upward) between levels can be acknowledged. The passage from level (i) to level (ii) seems to demand that at least the following three conditions be met: That a given body of conjecture (system of meanings, theory)

(1) be objectively learnable: 'so far as science, real science, extends,' Husserl tells us, 'so far one can teach and learn, and this everywhere in the same sense' (1911, p.73);
the theory must also be such that

(2) there is an intersubjectively accepted procedure for the justification of its individual propositions - for example: the repeatable experiment of natural science, (cf. 1911, p.74);
and finally the theory must be such that

(3) it possesses the cumulative character, the character of possessing at every stage an intersubjectively accepted theoretical basis the periphery of which is subject to a constant process of enlargement (with 'local' set-backs only), as new problems are discovered and new methods of solution are obtained.

For a 'special science', which meets these conditions there is an ever-present demand that the theory involved be raised to the level of a rigorous science. But this is not the task of the scientist (e.g. of the mathematician) himself, but of the philosopher. Husserl recognises very clearly that no amount of 'internal' scientific re-organisation can serve to raise a discipline to the level of a rigorous science (see LU § 71). Thus no mathematical consistency proof could serve to secure the foundations of
mathematics as a whole - and as a result of Gödel's incompleteness theorem we have a particularly clear insight into why this should be so. The task of 'external' reflection which is demanded is in fact precisely a philosophical task. Thus it is the philosopher alone who is the 'true theoretician', who can alone raise a given technical discipline to the level of scientific rigour. But Husserl also very clearly recognises that it is a task which cannot involve the philosopher himself becoming a special scientist manqué, for example by borrowing the methods of the specialist or by seeking to 'amend' his results. Philosophical investigation does not seek to meddle in the work of the specialist, but to achieve insight in regard to the meaning and to the nature of his achievements as regards their methods and their objects. (LU, 234; my emphasis cf. Findlay, p.245 of LI).

To withstand the 'epistemological criticism' of the philosopher and thus to enable the formation, at least in principle, of a correlated rigorous science, a given special science must be such that isolated fragments of knowledge can become unified in the coherence of a general theory (LU, § 6, cf. also Bolzano, 1837, §§ 1-4 for a more pictorial account of the considerations involved here). But more importantly it must be ensured that it is 'knowledge' with which we are dealing, and not some foundationless semblance of knowledge. For this it is necessary that the propositions assembled so far, which will form the basis of future scientific advance (for there is no suggestion that one could provide a rigorous foundation for a cumulative, growing discipline as a whole), that these propositions should be capable of being verified in thought, of being evidentially given as true of those states of affairs to which they would correspond. And it is necessary also that the arguments and proofs erected on the basis of these propositions should be equally capable of an evidential validation, resting on a double insight both into the validity of the arguments involved and in the possibility of a renewed validation on the part of those who would come after.

Interestingly Husserl recognised that even philosophy itself has
failed to meet these requirements, for though it may well be that 

the proposals presented in the great scientific works of philos-

ophy in ancient and modern times are based on serious, even
colossal intellectual activity. More than that it may in large
measure be work done in advance for the future establishement of
scientifically strict doctrinal systems; but for the moment,
nothing in them is recognisable as a basis for philosophical
science, nor is there any prospect of here and there cutting
out, as it were, a fragment of philosophical doctrine with
the critical scissors. (1911, p.75).

We do not go so far as Husserl, however, in his belief that the science of

phenomenology would provide the unique discipline which would raise
philosophy, and thus eventually all other admissible sciences, to the level
of theoretical rigour: as we shall see, there is more to the achievements
of extra-phenomenological philosophy - including many of the contributions
of Husserl himself - than Husserl, qua phenomenologist, would lead us to
believe.

What, now, are the consequences of these distinctions with regard
to mathematics, and then also with regard to logic? It is clear from
Husserl's writings that his own point of view, were he in a position to
survey the achievements of logic since Frege, would be to regard the 'positive'
logical discipline as closer, in the crucial respects, to the discipline
of philosophy, as something which would involve separate 'schools' within
which even the most basic principles of the remaining schools are called
into question. Both 'positive' logic (mathematical logic) and philosophy
would be held in an inferior position as compared with mathematics and
phenomenology, both of which would be regarded as truly rigorous or as
capable of becoming so. But there is something artificial in this opposition,
not only because phenomenology has itself failed to live up to the demands
which Husserl set for it, but also because mathematics is further from
'rigour' (by his paradigm) than Husserl supposed. For Husserl's paradigm of
rigour was one which involved the passage from a given body of special-
scientific activity relative to a fixed subject-matter, to a unique rigorous
discipline correlated with that activity. It seems further that Husserl
imposed this paradigm upon mathematics as he experienced it. For even
before Husserl turned to philosophy the notion of 'standard' mathematics
had become at least problematic. (This is shown by Tiel in Ch. 3. of his
1972). And the case of intuitionist mathematics reveals in a particularly
sharp form the possibility of alternative mathematical disciplines whose
results would be in conflict with classical mathematics, in a way which
would be reflected in any rigorous theory correlated with either of the
"regimes" involved. 52 That Husserl failed to react to the challenge
of intuitionism - even though Brouwer's works had gained wide currency
amongst philosophers in Husserl's day, and even though Becker's Mathematische
Existenz, which Husserl published in his own Jahrbuch, contains extensive
discussion of those works and of the Grundlagenstreit between intuitionism
and formalism - reveals that he was constrained to regard such a discipline
as something which would lie only on the fringe of 'true mathematics'.

Once this erroneous paradigm of rigour is removed, however, the
insights which Husserl achieved into the different levels of scientific
activity can be preserved in all their forcefulness. As a first step
in developing an amended paradigm and as a step, also, towards solving
our still unanswered questions concerning the nature of the theoretical
discipline of logic, it will be useful to pause, for a moment, to discuss
the relation of Husserl's views to those of Frege and his successors
in the analytic philosophical movement.
§ 9. Formal and Informal Rigour

Let us recall our question of pp. 55-6 above as to the extent to which either of the 'special sciences' of standard mathematics and mathematical logic can become rigorous sciences; that is, to what extent can the foundations of these disciplines be both ontologically and epistemologically secured? In this respect it is of the greatest interest that attempts were made by Frege in his Grundgesetze (Gg) and by Whitehead and Russell in Principia Mathematica (PM) to establish a foundation for mathematics in conformity with criteria identical in their effect with those laid down by Russell. The attempts failed, or were abandoned, but this was not because of any intrinsic deficiency of the programme of establishing mathematics as a rigorous science. They failed, rather, because they rested on principles admitted as 'rigorous' which were inadequate to the subject-matter involved, both in being insufficiently powerful and in being insufficiently refined. This combination of qualities is important in virtue of the fact that, as Frege's Gg system illustrates, a combination of power with insufficient refinement leads to inconsistency. Whitehead and Russell attempted to reverse this balance, but the (type-theoretical) 'refinements' to which they appealed were, as we shall see below, clumsy indeed from the point of view of ontology, and their role in the PM system is in any case dubious, in virtue of an appeal to the questionable 'Axiom of Reducibility' which was indispensable to Whitehead and Russell if the full power of classical mathematics was to be reproduced within the system.

With the failures of these early attempts one might have anticipated that foundation-theorists would have embarked on a search for more refined and more powerful, yet still 'rigorous' principles which would be more adequate to the complexity of the domain of investigation. Instead, however, the very notion of 'rigour', at least among many philosophers, was brought into disrepute.
Thus there has been a simplistic reaction of das Kind mit dem Bad auszuschütten (Wuchterl, 1967, p.222), expressed for example in Russell's confession, in his introduction to the second edition of PM, that the dubious reducibility axiom is admitted even though it has a purely pragmatic justification. It leads to the desired results, and to no others. (PM2, p.xiv).

This signals a change in conception of the philosopher's task which is still too little appreciated. For 'foundational' investigations, that is to say, investigations carried out with the aim of developing a foundation of, say, mathematics as a rigorous science, had hitherto been characterised by an attitude of faithfulness to the pre-given subject-matter, which had been supposed, not only by Russell but also by the earlier Russell and by Frege, to be unique. This attitude has been well-expressed by Geiger in his (1924) account of the method of Wesensaxiomaticik applied by Geiger himself to the problem of finding a rigorous formulation of Euclidean geometry. Geiger puts forward as the first Wesensaxiomatic principle for the 'discovery of axioms' the following

Principle of the correspondence of structure between the object-world and the axiomatic system:

Just as many fundamental concepts must be acknowledged within the system as there are fundamental elements and relations to be found in the structure of the object-world [the domain of research of the original special science]. Their number must not be artificially reduced through spurious definitions. Equally the deductive procedure must allow only those consequences and derivations to be made use of within the structure of the system which correspond to founding-relations und detachable complexities [Fundierungen and Komplikationen] on the side of the object-world.

With the establishment of pragmatic adequacy as the over-riding criterion of evaluation for foundational systems - something which has been encouraged by the increasing dominance of positivist and empiricist attitudes in philosophy, and which had received an initial impetus from certain characteristics of Frege's philosophy - Geiger's principle comes to be rejected.
In place of Geiger's principle which, as we said, characterises an attitude of faithfulness to the pre-given (ontological) subject-matter of the particular science under investigation, there arises an attitude on the basis of which the ontological aspects of a science, its 'object-world', fall out of the picture completely. Again, what is involved here in the general case can be illustrated by considering the case of mathematics. The two systems of PM (i.e. with and without the axiom of reducibility: the 'simple' and the 'ramified' theory of types) having been shown to be, for different reasons, inadequate to the pre-established subject-matter of classical mathematics, one did not go back to that subject-matter, armed with new insights, to see how things could be ordered differently in any 'rigorous science' of mathematics. Instead one regarded the two systems themselves as determining (intentional) object-worlds of their own. These counterpart object-worlds may be considered as the reflection, on the side of ontology of the axioms and rules laid down purely formally within the systems involved, somewhat as the purely intentional domain of represented objects of a novel may be conceived as the ontological reflection of the isolated system of sentences laid down in the novel. But given this new freedom to construct one's own mathematical object-world the problem of providing a foundation for pre-existing mathematics became usurped by the (different) problem of re-constructing mathematics from the ground up, with the consequence that inferior mathematical domains - in particular arithmetic (including ordinal arithmetic) - come to acquire an exaggerated philosophical importance. 58 Certainly this policy led to much valuable work being done. On the one hand the development of intuitionism by Brouwer and his school, and of the various constructivisms put forward by Weyl, Chwistek, Lorenzen and Wang. (This work has the effect of generating an ontologically 'new' mathematics but it is not carried out with this aim in mind. It is conceived rather as a continuation of the task of rigorous science but with exclusive attention to the problem of providing an epistemological foundation for mathematics, of securing the validity of the deductions involved, even if this is at the expense of the ontological foundation of mathematics as
practiced hitherto. There is an ever-present willingness to dismiss, as 'mere ornamentation', those higher reaches of classical mathematics which show themselves, by intuitionist or constructivist lights, to be incapable of an epistemological foundation). On the other hand there is the work of Zermelo (on the cumulative type hierarchy (1908)), of von Neumann on the theory of proper classes (presented, with modifications by Bernays and Gödel in Gödel, 1940), and of Quine (especially 1937 and 1951), work which is explicitly carried out as the development of a counterpart to classical mathematics, including e.g. the furthest reaches of Cantor's 'paradise' of transfinite ordinal and cardinal numbers. What must be noted is that each of these latter approaches is isolated from the task of describing any pre-existent object-world: instead they are called on to satisfy merely the pragmatic requirement that only the (useful) results of pre-existent mathematics be preserved. As we shall see throughout the present work, this requirement, and its equivalent in other spheres, leads to a degree of freedom in the ontological sphere which is intolerably great.

As a result of this change in the concept of philosophical 'foundations' a quite remarkable situation arose. For the same principles which had been shown, in the failure of the programmes of Gg and FM, to be inadequate even for the rigorous formulation of parts of mathematics, came to be applied outside the ontologically still relatively unproblematic domain of mathematical objects to the experiential world in general, and with identical consequences. Here again, because of the now even more pronounced shortfall of the machinery at hand to the task of coming to grips with the subject-matter under consideration, it became necessary to deal with constructed models of the world rather than with the world itself. Once again the attitude of 'ground-upism' led relatively inferior elements in our experience of the world to be ascribed an exaggerated importance at the expense of higher-level elements which constitute, for example, our intellectual life.
This is not to suggest that a completely adequate 'Aufbau', a system which would truly satisfy Geiger's correspondence principle, could not be constructed. Indeed there are close relationships - pointed out by Kl"{o}ng between Husserl's programme of phenomenological 'explication' and the programme of logical explication engaged in by Carnap, relationships which suggest that phenomenology may be seen as itself an attempt to produce such an adequate Aufbau. But certainly in order to attain such adequacy one would have to sacrifice the reductivist simplicity which has characterised those logistic systems which have so far been produced. It is perhaps 'only after the gigantic preparatory work of generations' (Husserl, 1911, p.76), through the stages of which ever more complex conceptual frameworks become accessible to each new generation of scientists, to be interiorised by the generations which follow, that an adequate Aufbau could be brought to realisation.

The most radical position concerning the supposed failure of the programme of providing a 'rigorous' foundation for mathematics - the programme of Wesensaxiomatik - was the formalist doctrine. This doctrine is one which holds that all ontological questions, and with all questions concerning the truth of mathematical statements, may be dismissed as metaphysical superfluities. The formalist holds that it is possible to conceive mathematics and logic as series of formal calculi which would in turn be conceived as systems of meaningless marks. Already in the first years of this century, when the formalist philosophy of mathematics was in its earliest infancy, Frege had mounted a thorough critique of its fundamental theses. He opposed 'formal arithmetic', arithmetic qua mechanical calculus, to what he called inhaltliche Arithmetik (contentual arithmetic) (see his 1908, esp.p.55, Eng.trans.p.137 and cf. S&R,pp30-33). The latter may be identified simply as standard arithmetic just as it has been traditionally practised since the time of the Greeks, at least in so far as (i) we confine our attention to the contentual arithmetic of finite numbers, and (ii) we exclude from consideration those informal arithmet-
ical practices which have themselves a purely mechanical or manipulative basis - however indispensable such practices may be for the processes of learning and applying 'rigorous' arithmetic. For whilst it is of course possible to operate with figures mechanically, just as it is possible to speak like a parrot: ... that hardly deserves the name of thought.

For to operate with figures in a mechanical way only becomes possible at all after the mathematical notation has, as a result of genuine thought, been so developed that it does the thinking for us, so to speak. (Frege, Gl, p.IV). It is this 'genuine thought', then, of which contentual arithmetic consists, and which provides the foundation both for practical, manipulative arithmetic and for such normative statements as: 'When you add two and two together you should get four' - and then also for such applied mathematical statements as: 'two apples together with two apples make four apples'.

What Frege recognised was that a purely formalist approach to arithmetic, which abstained from any appeal to contentual arithmetic as its standard in the construction of formal calculi, would be no more tenable than an approach to mountaineering which abstained from any appeal to the rock-face of the mountains which one was attempting to climb. (Cf. Frege's own 'creep' analogy on p.55 of his 1908).

For it is contentual arithmetic which can alone determine, for any given calculus, whether that calculus is a calculus of arithmetic', that is to say, a scientific calculus at all. Frege correctly argued that what alone 'raises arithmetic from a game to the level of science' (Gg,II, p.100) is its applicability in the widest sense, that is applicability not only in extra-mathematical fields ('2 oranges plus 3 apples is 5 fruits', etc.) but also within mathematics where it forms the foundation of higher-level mathematical disciplines, not necessarily in the sense that only through being 'reduced' to arithmetic can such disciplines be counted as truly rigorous, but simply in the sense that arithmetic forms a crucial building-block of such disciplines. As Thiel has pointed out, however, it is a
'matter of wonder', even today,

that Frege undertook simultaneously to solve the problem of foundations and the explanation of applicability ... by providing a logical model (obtained through an analysis of numerical propositions), (S&K,p.33, my emphasis).66

There is some doubt, however, as we shall see, whether he could have succeeded in this task.

First of all there is the doubt as to whether the domain of propositions selected by Frege was sufficiently large to enable foundations to be provided for the whole of mathematics which were ontologically adequate. The development (since Frege's death) of category theory, for example, which resists an analysis of the kind provided by Frege, suggests that a more general analysis is indeed required. (Compare the criticism of Wittgenstein's philosophy of mathematics intimated in note 58 above, and the remarks on Riemann in the introduction to Ch. 4 below).

Secondly, and here we return to the issues of the previous section, the analysis of the special science of mathematics which Frege provides is an analysis which is, in an important sense, carried out on the level of mathematics. As Husserl remarked, 'the sense and justification' of mechanical arithmetic and of positive mathematics in general 'depends on validatory thought' (IU,24; LI,69, cf. n.65 above). According to Frege this was to be provided by validatory thought internal to mathematics, that is to say by determining the extent to which 'positive' mathematics lives up to the standards set by 'contentual' arithmetic and by its sister disciplines for other branches of mathematics. According to Husserl, however, validation in the true sense could only be obtained on the more abstract level of philosophy and by quite other methods - described in the section which follows - than those used by the special scientist himself.

Perhaps the opposition between these two approaches can be expressed as follows: the term 'reason' is both a noun and a verb. Frege, like, for example, Plato before him, seems to have leaned heavily toward a view that
the former sense was logically prior, a view which Husserl, even in the
'logical objectivism' (Morscher, 1972) or 'logical realism' of the LJ period
was always careful to reject. One consequence of Frege's view was his
assumption of a determinate totality of 'logical objects', entities 'trans-
parent to reason' (Gl, § 105), the eternal and immutable relationships
between which were reflected in the propositions of contentual arithmetic
and other contentual disciplines. Hence, too, the latter are characterised
as constituent disciplines of logic, the science of reason per excellence.
The task of providing a foundation for mathematics became, for Frege, the
task of displaying these eternal and immutable relationships in a way
which would be in perfect conformity with the 'correspondence principle'
distinguished by Geiger (p. 61 above). But the development of a formally
rigorous 'super-mathematics', however great a technical feat this may
have been, and however much light it may throw on the nature of logic and
mathematics from the formal point of view, is insufficient from the
point of view of validation - as Frege himself recognised in his critique
of 'formal' arithmetic from the standpoint of his 'contentual discipline'.
As Thiel has written

the mere establishment of an axiomatic system for arithmetic
cannot rate as an establishment of arithmetic but itself needs
establishment (S&R, p.32f),

and this establishment, which would ensure that Frege's supermathematics
is a truly contentual discipline, can be achieved only on the level of
philosophy, on the level of reason as a continual, critical, elucidatory
activity, relative to the individual special sciences with their essentially
non-theoretical 'inventions'.

§ 10. The superstructure of the realms of meanings.

Frege's notion of 'logical objects' suggests a conception of logic as a science of a platonistically existing object-world. Such a conception has become untenable however, largely as a result of the collapse of the 'full' platonist approach in the foundations of mathematics. It has given way to an approach to logic as a 'free science' which would have no pre-determined subject-matter, but would consist only of (e.g. technically useful) 'inventions' of formal systems considered as object-entities, as symbolic structures. But there is an alternative to both of these approaches, which is the Husserlian conception of logic as a discipline whose subject-matter consists not of objects but of meaning-entities. (See LU, Investigation I, esp. § 29).

In § 7(2) above we pointed to the existence of certain relatively abstract 'superstructural' features which could be discerned in each of the domains of object-entities there distinguished. We intimated also that higher-level features of a parallel kind can be distinguished in the realms of meanings. In fact it is in virtue of these higher-level features that a truly systematic theory of meaning can be developed at all, given the infinite complexity of the domain of 'substantial' meanings as such. This domain consists of individual 'thoughts' (identifiable as 'propositions' when expressible by means of a complete assertoric sentence), of ideas and presentations, and it includes also more complex structures of thoughts, ideas, and presentations which, in combination with certain higher-order acts, yield higher-order substantial meaning-unities such as arguments, proofs, and even whole theories. But associated with such 'substantial' meaning-unities there are also what Husserl calls the 'elementary connective forms' (LU, § 67), that is to say, the logical connectives: 'and', 'or', 'if...then...', etc., of modern logic. The latter 'connect' propositions in such a way as to constitute them into arguments, proofs and theories. Then there are the 'propositional operators': 'it is true that...', 'it is not the case that...', 'it is necessary/possible/impossible that...', 'A knows/believes/doubts/denies/remember/assumes that...' which, when applied 
proposition yield a new, more complex proposition having a new set of logical
relations to other propositions (e.g. different truth-conditions).

And now this superstructural level of meaning-entities will contain also the
various argument-forms and theory-forms which are exhibited or instantiated
by 'substantial' arguments and theories as such. (LU, § 69). These 'forms'
are higher-order meaning-entities which can, however, never be actualised
as such, in the way that arguments and theories are themselves actualised
in the thinking activity of, for example, scientists and mathematicians.

Concepts which relate to this superstructural level of the realm of
meanings will include the concepts of logical consequence, the concepts of
validity and invalidity of arguments, the concepts of rigoroussness of
proofs, and the concepts of logical consistency and logical completeness of
whole theories. For each such concept has application to the higher-order
actualised meanings involved only to the extent that it has application to
the higher-order meaning-forms which are instantiated. Thus we can affirm
that such concepts - the concepts of logic - are concepts which have their
primary application to those meaning-forms and, more generally, to all the
entities which make up the superstructural level of the realm of meanings.

This suggests that we acknowledge logic itself as a discipline whose subject-
matter consists precisely of this superstructural level of the domain of
meaning-entities. It follows that logical laws are first laws in the realm of
sense which only mediately relate to reference.

Such a conception explains how logical laws are in a certain sense al-
ready known to every thinker, to every theorist; for thought in general and
theoretical thought in particular presuppose a grasp of the order and articu-
lation of meanings (though not necessarily of linguistic meanings alone)67,
and in obtaining such a grasp the thinking subject thereby attains a hold
upon the structural laws which pertain to meanings as such. (Compare Tract-
atus, 5.4731). This in turn explains how conceptions of logic as a scient-
ific discipline can escape the charge of vicious circularity. Consider, for
example Bolzano's definition of logic (the 'theory of science') as
the science which instructs us in the representation of sciences in adequate treatises. (1837, §1).

Or Husserl's wider definition of logic as
the technology of scientific knowledge which sets up rules for the demarcation and construction of the sciences (LU, 29; LI, 73 - here 'science' has to be interpreted in terms of Husserl's concept of 'strange Wissenschaft' sketched above).

As Bolzano himself points out, according to definitions of the given type, logic
is to teach us how the sciences ought to be represented, and since it is itself a science, one might ask how it can be produced, since one does not know how any science is to be represented as long as there is no theory of science. (1837, §2).

This doubt can be easily removed however for one can proceed according to the rules of logic and thus generate any number of sciences (or rather their written expositions), among others logic itself, without being clearly aware of these rules, (loc.cit.)

Just as an artist may create beauty without knowing how, and certainly without making any explicit recourse to the rules of aesthetics, so a scientist, and in particular a logician, may conclude an investigation without recourse to the rules of logic. (LU, §4).

The analogy with aesthetics is important also in that it indicates the way in which logic, or the grasp of its laws, is something which can be only partially or faltering acquired - just as the conditions for producing successful works of art may be only partially met by a given artist or a given school. And it indicates also the way in which the production of individual arguments, proofs or theories - as of individual works of art - may be a gradual, heuristic process, perhaps involving several attempts, any one of which may be accepted as valid or as consistent or complete at a given stage only to be rejected later as a result of deeper reflection or of new awareness.
The analogy with aesthetics would be a false one, however, if it was held to suggest that the laws of logic are in any sense subjective or culture-sensitive, or that they may be expressions of mere fiat or of agreed or imposed convention.

Note that the conception of logic as a discipline whose subject-matter is to be sought in the superstructural features of the domain of meaning-in-general by no means implies that logic is to be comparable in any sense to 'positive sciences' of linguistics or of grammar (which have their own 'rigorous' sciences of pure meaning-categories quite distinct from the 'rigorous' science of logic). Nor, in the development of the discipline of logic is it necessary to select some suitably-developed language and 'read off' its superstructural features. For if logic is to be an adequate theoretical basis for a philosophically justifiable normative discipline (of the type discussed in § 8 above) no factual investigations can serve to provide evidence for the theses which it involves. This applies whether these investigations would be directed toward languages (ordinary languages or artificially developed 'ideal' languages) or toward thought as actually executed. Such evidence can be provided, as both Russell and Frege in their different ways recognised, only by investigations of a sui generis ideal world. This is not to say that investigations carried out on the basis of our actual thinking activity would be of no help in the discovery of those relations which obtain in that world, the evidence for which could then be sought elsewhere, but that, as we shall see in § 12 below, is another story.

The laws of logic are, Frege tells us, as milestones set in an eternal foundation, which our thought can overflow, but never displace. It is because of this that they have authority for our thought if it would attain to truth.

Thus they certainly do not bear the relation to thought that the laws of grammar bear to language; and nor do they make explicit the nature of our human thinking and change as it changes. (Gg.,I, xvi, cf. Eng.ed.p.13).
There is nothing in this conception which need come into conflict with our claim that logical laws are 'first laws in the realm of sense', for as we shall see in § 17 on 'Platonism' below, there is an important sense in which a platonism of 'meaning-entities' is more acceptable — and more explanatory than — a platonism of objects. Our claim does however seem to be flying in the face of Frege's other views concerning logic which we have considered, to the effect that logic is a discipline whose subject-matter consists of certain logical objects and in particular of the two truth values, truth and falsehood.

The word "true" [Frege tells us] indicates the aim of logic as does "beautiful" that of aesthetics or "good" that of ethics. (Ged, 58; Klemke, ed. p. 507, compare the quote from Husserl on p. 54 above).

The unity of Frege's position is further reinforced by his conception of numbers as logical objects and of arithmetic as a sub-discipline of logic, a position which surely excludes, we might suppose, any view according to which logic is a discipline which relates solely to meaning-entities. Such a view is, however, capable of being made at least consistent with Frege's position, as we shall see.
There seems to exist a conflict in the conception of logic as a science of meaning-entities (superstructural meaning-forms), whose subject-matter yet consists of 'entities of reason' which are characterised by Frege as logical objects. This conflict could be resolved, however, if we could establish that for Frege the term 'object' can be applied also to certain entities which belong to the realm of senses or meanings. And this is indeed the case, for although there is some dispute as to whether Frege regarded (or ought to have regarded) the meanings of unsaturated (incomplete) expressions as being themselves unsaturated, certainly in the case of saturated, object-donating expressions Frege was very clearly of the opinion that the associated meanings were themselves saturated, i.e. that they were 'objects'. (See Angelelli, 1967, p. 50).

In contrast to the treatment of Fregean concepts and functions, the objects (Gegenstände) of Frege's ontology have too often been taken to be unproblematic, even given Frege's inclusion of e.g. truth values in the category of objects. Russell, for example, writes that whilst Frege's use of the word Begriff does not correspond exactly to any notion in my vocabulary... On the other hand, his Gegenstand seems to correspond exactly to what I have called a thing. I shall therefore translate Gegenstand by things (1903, p. 505).

There is, however, a need for a more refined interpretation, one which would recognise that there are many instances where by 'Gegenstand' Frege does not mean 'thing' at all. (See § 17 below). What are excluded from Frege's ontology are simple 'incomplete' or 'unsaturated' objects, objects which would be denoted by conceptual or functional expressions and not by proper names. We are suggesting that even if there is a valuable ontological distinction to be made between 'objects' (Gegenstände) - in Frege's sense - and functions (including Fregean concepts), then it ought to be possible to present this distinction within a framework where the meaning-object distinction, which is in our eyes more basic, is taken into account.

In particular this would demand that we adjust Frege's terms in such a way that
no meaning-entity is admitted as an object. The argument is that it is possible to make this adjustment without departing substantially from the coherence of Frege’s underlying position, but in such a way that his domain of ‘logical objects’ — including numbers — comes to be interpreted as precisely the domain of superstructural meaning-entities isolated above. For logical ‘objects’ are presented by Frege as entities which, like meanings generally, serve as that through which we order and articulate our experience of the individual objects-proper (using ‘object’ now in the strict sense) which make up the world of ordinary reference. This is true even in the case of numbers, for

in arithmetic we have to do with objects which are not given to us as strangers from the outside through the senses, but with those which are given directly by reason which sees through them as its own (welche sie als ihr Eigenes völlig durchschauen kann) (Frege, Gl, § 105, as trans. in Thiel, S&R, p.65, and cf. Moss,(4))

Further support for the conception of Fregean logical ‘objects’ as meaning-entities is provided by the passages in which Frege lends his qualified support to the programme of Inhaltalogik (the logic of conceptual contents). The Inhaltalogiker presented a defence of the view of logic as a science of meaning-entities, in opposition to the Umgangalogik (logic of extensions) of Schröder and his school, which attempted to augment the techniques of the Boolean domain-algebra in such a way that correlates of the traditional higher-level logical notions could be developed.

Schröder had complained that despite many thousands of years at work, philosophers were still far from in agreement concerning the way in which one could even begin to develop a unified theory of concepts which would explain in a general way both the nature of concepts and their manner of formation.(Cf. his 1890,p.98). Thus, he argued

it would be an unscientific procedure to want to erect on the basis of such an insecure and disputed foundation a science which, like logic, would put forward the claim to establish only absolutely secure because epistemically necessary denknotwendige and evident truths. The project of Inhaltalogik of erecting logic as such a
Science of conceptual contents may well be compared with the attempt to build the roof before the house which would support it. (op.cit., p.99)

Schröder held that it would be possible to provide a sure foundation for logic on the basis of extensions of concepts, but in order to escape the charge that even then he would be presupposing the existence of an adequate theory of concepts, he found it necessary to treat such extensions or 'classes' as unarticulated 'domains', *totalities of things* (conceived on the model of aggregates or heaps). Thus, in particular, the class consisting of one single thing as its only element is counted as identical with that thing itself.

Frege, in his review of Schröder's *Algebra der Logik* (Frege, 1895), was able to point out a whole range of inconsistencies and ambiguities into which Schröder fell as a result of his attempt to found logic, the science of articulation *par excellence*, upon a basis within which articulation was left out of account. Thus for example on Schröder's introduction of an empty class - an object whose status can only be that of an intellectual construct - Frege can justly argue that a class *qua* 'domain'

consists of objects: it is an aggregate, a collective unity, of them; if so, it must vanish when these objects vanish. If we burn down all the trees of a wood, we thereby burn down the wood. Thus there can be no empty class. (1895, p.436f).70

For what happens on Schröder's conception when we 'remove' the 'elements' from a class is that the class itself also disappears.

A related criticism of the conception of a class as a domain or totality of things is that whenever, in logic, we wish to state something about a class, e.g. that it is included in a more inclusive class, we should then willy nilly be asserting something about all the objects which make up the totality in question. But when I utter a sentence to the effect that, say, the class of men is included in the class of mortal beings, that is when

I utter a sentence with the grammatical subject 'all men', I do not wish to make an assertion about some Central African chief wholly unknown to me... It is likewise equally false that in such a sentence many judgments are put together by means of the common name, as Herr...
Schröder thinks (1890, p.69). In order that a word like 'man' or 'planet' should have logical justification, it is necessary only that there should answer to it a sharply delimited concept; whether the concept comprehends something under itself is not here relevant. (1895, p.454).

How, then, can Frege form a conception of classes if not as totalities of elements, and what role does class-talk come to play in Frege's logic?

It becomes possible to express general thoughts when one states relations amongst classes, Frege writes, and thus to come to the level of logic, only when classes are determined by the properties that individuals in them are to have, and because we use phrases like 'the class of objects that are b' (1895, p.452f).

The classes which then result are no longer totalities of things, but, to use Frege's term, they are value-ranges \( \text{WertverlMufe} \) of associated functions (including, of course, concepts). One sometimes finds the suggestion that these value-ranges are to be compared with graphs in mathematics. For example the value-range of the cube function is to be conceived on the model of a two-dimensional graph: along the 'x'-axis we have, first of all, positive and negative numbers, the cubes of which are allowed to determine coordinate pairs \((x,x^3)\) against the 'y'-axis. But also along the x-axis we must conceive of there being represented all other objects, including the moon, the sun, the true, the false, all other \text{WertverlMufe}, Frege himself, and Dr Gustav Lauben. Each of these positions on the 'x'-axis is correlated with some arbitrary selected point, say, 0 in the 'y'-axis - and must be so correlated if the given function is to satisfy Frege's (peculiar) requirement that all functional expressions, if they are to denote functions, must be defined for all objects as arguments. In the case of (those functions which are) concepts, the 'y'-axis consists merely of the two positions correlated with 'the true' and 'the false', but the 'x'-axis, again, consists of coordinate points correlated with each and every object in the universe. If one can accept this model at all, of course, then the temptation is to go one step further and appeal to the (useful) set-theoretic conception of n-dimensional graphs as sets of
ordered n-tuples. It is difficult (perhaps even for pedagogical reasons) for the modern logician to grasp the extent to which Frege had something different in mind for his Wertverläufe. Yet if this is not grasped it becomes impossible to understand, in turn, how Frege could have been able to insist on the requirement that functions are defined for all objects as their arguments, a requirement which, I believe, makes a mockery of the graph-theoretical model.

Wertverläufe are, first of all, entities of reason. It is to Wertverläufe quite generally to which Frege is referring (cf. the quote on p. 74) as entities which 'are given directly by reason which sees through them as its own'. We have suggested that all such 'transparent' entities of reason must be counted as belonging to the category of meaning-entities, and if concepts, too, can be counted as belonging to this category, then it becomes tempting to suggest an identification of a given Wertverlauf with its associated concept—and similarly for functions in general. Such a view would receive support from the suggestion of Sluga that

The Wertverlauf is in truth nothing but the function seen anew, i.e., the function treated as object. This difference between function and Wertverlauf is more of a difference of aspect than a difference of object. (Sluga, 1962, p. 202, as trans. in Thiel, S&R, p. 67).

Thus the difference between function and Wertverlauf would be an epistemological matter, corresponding to no differences on the side of the entities themselves. This view is further reinforced by the fact that, for Frege, to the extent that he was prepared to speak of identity of functions at all, functions and Wertverläufe are held to have identical identity-conditions:

If one talks about the identity of functions, one can only mean the identity of their Wertverläufe, or the identity of something which is univocally related to them. (letter of Frege to Russell, Aug. 3, 1902, as quoted by Thiel, S&R, p. 67).

What Frege could have meant by the caveat here we shall need to return to below. Now, however, we must consider the arguments, especially as assembled by Thiel, against the possibility of such 'Janus-like entities' (S&R, p. 68) as are countenanced by Sluga.
Commitment to hybrid Wertverläufe/functions has an initial consequence that Frege's distinction between saturated and unsaturated entities would disappear. But our identification of concepts as meaning-entities reveals a second drawback of Sluga's proposal. For Wertverläufe, no matter how distanced they may have been from objects which fall under the associated concept, were nevertheless conceived by Frege as extensional entities. That is to say, an identity of objects falling under two concepts implies an identity of Wertverläufe. Whilst Fregean concepts are so conceived that for them, too, it is possible to define an analogue of identity such that identity of Wertverläufe would in turn imply 'identity' of concepts (cf. Angelelli, 1967, p.206f) this is not at all possible when concepts are identified as meaning-entities. For it would imply an identity of all concepts associated with the two predicates: 'first continental land mass West of Spain' and 'first continental land mass East of India', including the concepts associated by Columbus with these two expressions which were, of course, importantly non-identical.

A more adequate account of Wertverläufe against the modified 'Fregean' background is to view them as object-entities correlated in some systematic way with concepts (and eventually also with meanings in general). In this light, as we shall see, there is an important comparison which can be made between Wertverläufe and Meinong's 'incomplete objects'; (one might identify the former as extensional incomplete objects, noting that the 'incompleteness' involved is only tenuously related to Fregean incompleteness - see pp. 316-19 below). This is not to suggest that Wertverläufe (or Meinongian incomplete objects) are ontologically acceptable? For there is clearly no limit to the range of object-entities which can be systematically associated with individual concepts. (One could, e.g., read a novel as a novel 'about' its characters, as a novel about the German general staff in World War II, and then also as a novel about one's friends and neighbours, who bear, at least in the eyes of one particular reader, an
uncanny resemblance to the German general staff, etc.). This suggests that
the notion of a unique Wertverlauf of a given concept is an illegitimate
one, and it may be that the principle flaw in the Gg system can be seen in
this light: that Frege assumed that he had found a way in which a unique
object-entity could be associated with each and every concept (function,
functional expression) within the system, where this was not in fact the case.
(Cf. the valuable discussion in S&R, Ch.4).

More generally, Wertverlauf, like concepts, seem to have been endowed
by Frege with an incompatible combination of epistemic or 'rational' accessibil-
ity, and of ontological autonomy. Both are projected as existing in a realm
which is detached from the domain of intellectual constructs, yet both categ-
ories are such that the examples given by Frege are all clear-cut cases of
such constructs. 73

Let us now return to our problem of defending an interpretation of Fregean
'logical objects' - including Wertverlauf - as meaning-entities. One line
of defence we selected - the view of Frege as an incipient Inhaltlogiker-has
been shot down; not, indeed, because Frege is seen as belonging to the
Schröderian camp, but because his (dubious) Wertverlauf uncomfortably
span the divide between content and extension. Thus we find Frege himself
arguing that the Umfangslogiker

are right when, because of their preference for the extension of a
concept to its intension, they admit that they regard the reference
of words, and not their meaning, to be essential for logic. The
Inhalts-logicians remain only too happily with the meaning...

they fail to
consider that in logic it is not a question of how thoughts come
from thoughts without regard to truth value, that the progress
from meaning to reference must be made; that the logical laws are
first laws in the realm of reference and only then mediately
relate to meaning (1892-95, p.134; as trans. by Mohanty, 1974;
p. 60f. Cf. also Frege's 1895, p.106).
Thus we seem to have come full circle, no clear decision having been reached concerning the possibility of a coherent interpretation (or modification) of Fregean philosophy within which logical 'objects' are recognised as meaning-entities. To establish our claim that such a position can be coherently maintained, it would be necessary to go beyond Frege's programmatic descriptions of his philosophical method and to give detailed re-expressions of his individual analyses in the terminology of the meaning/object dichotomy. In carrying out this exercise we should be justified in excluding from consideration only those elements of Frege's philosophy - such as his theory of indirect reference and his account of the referents of sentences as truth values rather than states of affairs, - which can be shown independently to be unacceptable (products, perhaps, of the mathematician Frege's Drang nach Verallgemeinerung.) Here we shall be able to attempt, in the chapter which follows, only the beginnings of such an exercise for Frege's philosophy of number. A useful preliminary to this task will be a brief discussion of the philosophical background against which both Frege's and Husserl's work in the philosophy of mathematics was produced: the background of logical psychologism.

(1) 'Cognitive' Philosophy before Husserl.

Husserl developed into a truly workable philosophy the insight into the indispensability of a validation of any argument which would purport to be rigorous, a validation which was to be achieved by ensuring that the argument in question is capable of an evidential enlivening in precisely coordinated acts of thought. But there were earlier attempts in the direction of a philosophy of logical rigour which would base itself upon 'cognitive' acts. These were made by a group of thinkers working, largely in Germany, on the borderlines of philosophy and psychology throughout the 19th century; here we shall be interested only in those amongst such thinkers who positively espoused views which would be called 'psychologistic' as this term is used today, that is, to imply commission of the so-called psychologistic fallacy, which consists in regarding the evidence for philosophical or logical truths as residing in the domain of empirical psychology. Stated in this blunt form the fallacy threatens such absurd consequences as that logical and mathematical theorems should be 'proved' by means of empirical investigations of the way people think, as Husserl remarks

the mathematician would merely smile if psychological laws were pressed upon him as supposedly providing a better and deeper rational grounding \(\text{\[Begründung\]}\) for his theoretical pronouncements. (LU, 169; cf. LU, p. 178f.)

What is little realised is the extent to which each of the major thinkers who subscribed to a broadly psychologistic position — and here we may mention Mill, Wundt, Sigwart, Erdmann, and Lippe — made deliberate attempts to incorporate sophistications into psychologistic theory which would enable the absurd consequences of the fallacy to be avoided. Without those sophistications, which made psychologism a doctrine much less alien to the philosophical tradition than, through its caricature, it now appears, the movement could not have achieved the position of dominance in German philosophical circles which it did achieve, and which it maintained until the time of Frege and Husserl.
Such sophistications are, with perfect justice, of no interest to the modern philosophical community, however, since we now know that the tempting premise which promoted them: that actual thinking activity can provide evidence for philosophical or logical truths, is untenable. There is a contemporary philosophical importance in the more general approach of psychologism, however, since consideration of the issues which are raised by this approach can bring into light problems and ambiguities in modern accounts of logic which are hardly noticed by current workers in the field.

This historical importance of psychologism is further magnified, of course, by the fact that it inspired a reaction on the part of two such important thinkers as Frege and Husserl, a reaction which was sufficiently hostile to lead them both to formulate explicit alternatives to the psychologistic doctrine. But an adequate historical account of this reaction has rarely been presented. It has, for example, been too often suggested, particularly by analytically-minded philosophers, that Husserl began his philosophical career in the works *Über den Begriff der Zahl* (UBZ) and *Philosophie der Arithmetik* (PDa) as a 'psychologistic' thinker in the sense determined above. It is also too often suggested that the move to an explicitly anti-psychologistic position in the 1st volume of *LU* was sparked by influence deriving from Frege, and in particular from Frege's polemical review of PDa.\(^76\) A similar claim has also been too frequently made in connection with the insights at the root of Husserl's two major doctrines in the theory of meaning, namely

1. the conception of meanings as universals (ideal entities), advanced in *LU*, (see §25(2) below),

and

2. the conception of meanings as 'entities through which' the subject articulates his experience and which, unlike universals, are capable of being themselves meant e.g. in an act of serious utterance, a view advanced in *Ideas*, (see Ch.6 below).

Recent work suggests however that only with regard to the latter case, where truly remarkable parallels can be discerned between Husserl's theory and Frege's theory of sense and reference, can it be convincingly established that there
has been influence on Husserl by Frege. With regard to (1) it seems that the two thinkers were working independently at the time when the crucial discoveries were made. But here also there are philosophically important parallels, and psychology, we suggest, was the principle factor which led to this coincidence of views. Let us now, however, investigate the (cautionary) philosophical importance of psychology in its own right.

The problems which exercised the psychologists concerned the relations between objective configurations such as linguistic meanings and logical laws, and the subjective experiences in which these are actualised. The key to understanding psychology is to understand the extent to which its adherents were (rightly) impressed by the broad conformity of our actual thinking and arguing to logical laws and by the wide intersubjective communicability of ideas through language. They were concerned to capture the 'regularity' of correspondence between the two dimensions involved within a philosophical theory, but - and here is the surprise - they held that this could be achieved by identifying the regularity involved as statistical regularity! The laws of logic come to be identified as statements of empirical psychology, which yields both a particular view concerning the evidence of these laws, and also a view as to their nature. The doctrine also supplies an account of the nature of the discipline of logic as a whole as 'not a science separate from and coordinate with psychology,' but rather,

to the extent that it is a science at all, it is a part or branch of psychology, distinguished from it on the one hand as the part is from the whole, and on the other hand as the art is from the science. (Mill, 1878, p.461)

It must not be thought, however, that logical psychologism was exclusively a preserve of the empiricists. Sigwart, for example, advanced a view according to which the laws of logic are not to be established by reference to empirical psychology, and thus in the end to actual conscious subjects, but by reference to an 'ideal consciousness in general,' to the 'movement of thought' of a 'perfectly constituted subject'. The problem with such a view is that we find it difficult to pin down any positive consequence which would
serve to differentiate it from 'empirical' psychologism. One is tempted to suppose, with Husserl, that if there is such a difference and if the 'ideal consciousness' does succeed e.g. in determining a somehow more adequate system of logic, this can only be because the ideal consciousness has itself been 'constructed to fit' that system (LU, 100; LI, 127). (Compare the argument appended to p.9 above). What is in the end no more than a pale (though 'personalised') reflection of a given system of logical laws constructed retrospectively could not somehow provide them with any kind of prior evidential grounding. A second important argument against Sigwart, which was also made by Husserl, is that

the constant reference to an ideal consciousness makes us feel, disagreeably, that the logical laws perhaps only strictly hold for the fictitious ideal cases, and not for such as we encounter in experience. (LU, loc.cit.)

What was valuable in psychologism - a recognition of the importance of thought and of what is given in thought for the development of a rigorous philosophical theory, has therefore been missed by Sigwart who has turned away from thought itself in the direction of some 'ideal' and unattainable surrogate. The ideality of logic is not to be understood in a Kantian sense as involving an 'Ideal of perfection', but as something which is itself given in thought, through what Husserl somewhat misleadingly called a 'categorial intuition' by means of which we experience the sui generis ideal concepts and relations which constitute the subject-matter of logic. Husserl correctly and highly insightfully saw that this categorial intuition was in many respects perfectly parallel to the sensual intuition by means of which we perceive physical objects, but what he failed to recognise, as we shall see, is the extent to which both categorial intuition and ordinary perception yield results which are problematic in certain crucial respects.
§ 12. Frege, Husserl and Logical Psychologism (Appendix to Ch.2.)

(2) Antipsychologism

The position of influence of psychologism through the latter half of the 19th century was not achieved except in the face of criticism. This came from two sides: on the one hand from members of the neo-Kantian school, especially Natorp, who defended the Kantian conception of logic as an *apriori* discipline. On the other hand it came from a series of thinkers who had been influenced by Leibniz rather than by Kant. Here we may mention Herbart, Bolzano and Lotze, all of whom put forward elements of what we should now call a *platonist* conception of logic and of mathematics. The influence of such thinkers was, however, confined largely to the Austrian world. Further, only in the writings of Bolzano were platonist elements moulded into the framework of a homogeneous theory, but it took some time, as we shall see, before Bolzano's work became generally understood and appreciated.

Although each of the above thinkers had exposed weaknesses of the psychologistic position, it was the combined influence of Husserl and Frege which was to bring the fall in influence of the psychologists and to persuade former adherents, such as (Theodor) Lipps, to repudiate their former stand. But the lessons which are to be learned from the attack which Husserl and Frege mounted against psychologism are in danger of being lost. This is because the progress in the philosophy of logic since the time when this attack was made is progress which has been achieved against an exclusively anti-psychologistic background. Thus *anti-psychologism* as such is no longer defended by explicit argument, rather it is taken for granted by all workers in the field. This has had the unfortunate consequence that attention is no longer paid by philosophers to those relations - between the 'objective known' and the 'subjective knowing' (cf. LIII, A VII; LI, 42) - relations which had seemed so important to the members of the psychologistic movement. These relations generate what Willard has called the 'paradox of logical psychologism', namely that
'logicians' truths' are, in some very important sense about and applicable to such particular events in personal careers as conceivings, asserting, and inferring, but they nonetheless, as Husserl, Frege, and others have shown, do not draw their evidence from the examination of things of that sort. (Condensed from p. 94 of Willard's 1972).

Frege neatly side-stepped the paradox by moving from what we might call subjective or 'natural' cognition - with its associated 'natural' language - to an objectified 'reason' with its associated Begriffsschrift (conceptual language). It is from 'reason', which is dissociated from any temporal acts of thinking, that the evidence for Willard's 'logician's truths' is to be obtained. But clearly there is some difficulty in determining how we gain access to 'reason' and its 'associated 'realms of senses', (even given those features of Frege's philosophy which point to an attempt by Frege to meet the psychologists on their own ground: for example the identification of sense with cognitive value, and the deliberate use of the term 'thought' to designate the (rational content of the) sense of a sentence (Ged, 60f; Klemke, ed.p.511), where other writers, with less proximity to the psychologists, have preferred the term 'proposition' and its equivalents.

In arguing the case for his approach via an idealised or objectified reason Frege was often guilty, as we shall see, of overstating the case against 'natural' cognition as a source of philosophical insights. Largely as a consequence of Frege's polemic (see n.82 above) a stigma has come to surround all psychological investigations in logic. Thus the considerations pertaining to the 'actualisation' of logical principles and logical systems - and of arguments and theories in general have fallen out of view. In this way analytic philosophers have ceased to be aware of the rich variety of problems which are presented in this area and have come to assume, following Frege again, that this field is one in which appreciable results cannot be achieved by the philosopher. For Husserl, however, no such turning away from the problem of actualisation was possible. As Tugendhat has noted, for Husserl

the strict separation of the ideal logical from the psychological
does not make impossible a specifically 'psychological' clarification of the logical - rather it precisely demands such a clarification.

(1967, p.16).

For what is already as it were built into the foundations of psychologically accounts of logic must be provided by the anti-psychologist in the form of a supplementation of the central theory of logical evidence upon which his account rests. This supplementation must provide a clarification - so conspicuously absent from Frege's philosophy - of the relations between the ideal laws of logic and our actual thinking, and the success of this clarification is to be measured according to its closeness of fit with that which is actually given in thought. Those anti-psychologistic theories, such as the various extreme forms of platonism, which resist such supplementation, thereby show themselves to be inadequate. (And here we may remark that the extent to which analytic philosophers' accounts of logic pass this test is at best in doubt, for so little has been done to relate such accounts to our cognitive experience. Dummett's attempt, for example, to remove the inscrutability from Frege's philosophy of reason by means of an analysis of language, is rendered valueless by the fact that 'natural' cognition is itself given an analysis which rests on the presumption that the use of language is somehow well-understood.)

The crucial point in any criticism of logical psychologism is to recognise that the psychologists failed to make any distinction between such 'clarificatory' investigations and investigations relating to the justification of or the evidence for logical laws. Clarificatory investigations continue to be indispensable, however, for it is in virtue of such investigations alone that logical laws can acquire their significance in actualisations. To put the matter in a perhaps oversimple way: actualisability would seem to have the same determinative role in logic as applicability has in mathematics. (Cf. p.65f. above). Noone would suggest that the evidence for mathematical propositions is to be sought in their application, and yet those mathematical propositions which have no application are rightly dismissed as having the status of formal 'games'.
§ 12. Frege, Husserl and Logical Psychologism (Appendix to Ch.2.)

(3) Frege and Husserl

Already by 1884 in the introduction to Gl, but most importantly in the 1st volume of Gg (1893), Frege had shown that he was in possession of the crucial insights to the detriment of the psychologistic position. Frege’s anti-psychologism was further established in his 1894 review of Husserl’s PdA, Frege adopting the latter work as his ‘psychologistic’ stalking-horse - with little justice as we shall see below. Certainly Husserl was familiar with this and other works of Frege at about this time, and Frege’s arguments may have contributed to some change in Husserl’s position with regard to psychologism; but the general philosophical approach advanced by Frege had little influence in the wider philosophical community. This was partly in virtue of the employment by Frege in his logical writings of a baroque (though in practised use highly flexible) symbolism, which was simply inaccessible to the psychologically-minded logicians of the day. But it may also have been because of what must be recognised as the carping, hit-and-run style evinced by Frege’s polemical writings (and not least by his review of PdA). It is therefore possible to conclude that it was Husserl, with the publication in 1900 of the 1st volume of LÜ - whose expansive and scholarly manner are the complete antithesis of Frege’s polemical style - who must be given the credit for establishing anti-psychologism as a viable philosophical position. This work made possible a decisive change in the philosophical climate and contributed to the ‘rediscovery’ of thinkers such as Bolzano (and eventually even Frege himself) who had anticipated many of the anti-psychologistic arguments which are to be found in LÜ.

It is often suggested that Husserl came to develop these arguments only after himself having gone through a period in which he subscribed to the doctrines which he so vehemently criticised in LÜ. Such suggestions claim their support e.g. from the passage at the end of the Forward to LÜ, I (LÜ A VIII; LI, 43), where Husserl defends the ‘frankness’ of his critique of
the psychologists with Goethe's remark that there is nothing to which one is more severe than the errors that one has just abandoned. But this passage in no way constitutes an admission on Husserl's part of having held, in his earlier works (especially PdA and UBZ) a naive psychologistic position i.e. a position involving commitment of any form of the psychologistic fallacy. For nowhere in those earlier works do we find any confusion between the problems of justification of logical laws and clarificatory investigations' (p.37 above) of their actualisation. Indeed the most reasonable interpretation of these early works, one which agrees substantially with Husserl's own account of them at the time of Lr (V-VIII; Lj,41-3) and which in no way denies the appropriateness of the reference to Goethe - would be to conceive them as early exercises in just the kind of 'supplementary clarification' already described: One has assumed that Husserl's 'Goethean error' was to commit the fallacy of psychologism, when that error was rather the assumption that empirical psychological investigations could be of use in such an exercise of clarification, an exercise which in itself is not only unobjectionable but even indispensable. For empirical psychology is limited in its scope to subjective, temporal properties of acts in a way which implies that it could never be adequate to the clarification of those higher-level objective, extra-temporal "contents" of acts which constitute the subject-matter of logic. What Husserl had not immediately recognised was that for such clarification to be possible it was necessary to move beyond psychology, that is to the level of philosophy as such - and it may be unfortunate that Husserl drew as a consequence of this recognition the idea that a new philosophical discipline, which he called 'phenomenology', was needed for this task.

We can now propose in a precise form the question: what was responsible for Husserl's recognition of the inadequacy of empirical psychology to the tasks of clarification of logical (and mathematical) principles? The orthodox answer to this question, which finds its definitive exposition in Føllesdal's monograph (1958), is that the crucial role was played by Frege's review of PdA. This review was published at just about the time when, according to the
textual evidence which was until recently brought to bear on the matter, the change must be presumed to have occurred. This view has become less tenable since the recognition by Mohanty of elements of what can only be characterised as an anti-psychologist theory of meaning in very early works of Husserl from some years before the appearance of Frege's review (and also before the appearance of Frege's SuB). Mohanty's work, which we shall discuss in our chapters (5 and 6) on the theory of meaning below, casts doubt upon the Føllesdal view, since certain aspects of Husserl's later theory of meaning had seemed so decisively Fregean in origin. Here we shall confine our attention to the successive logical doctrines which Husserl shaped and advanced in the crucial period, and in this our task is greatly aided by the thorough chronological survey of Husserl's writings and academic activities which Holenstein has prepared as part of his Introduction to the Huserliana edition of the 1st volume of LU.

Holenstein's investigations reveal the fluid nature of Husserl's thought, the character of reculer pour mieux sauter which it exhibits during the period when the breakthrough to 'phenomenology' was made. Thus he points out that Husserl, in his "Psychologische Studien zur elementaren Logik" of 1894, with reference to PdA still defends the significance of psychological studies of the elementary processes of intuition and representation for the "a priori" disciplines. But nevertheless the possibility of being able to advance logical understanding of symbolic thought even without psychological research into foundations is expressly admitted. (Holenstein, 1975, XXII, my translation here and below).

In 1896 Husserl delivered in Halle courses of lectures on logic and the problem of psychologism containing important preliminary workings for the eventually published Prolegomena to Pure Logic (i.e. vol. I of LU) and including certain anticipations of central anti-psychological arguments. Husserl even claimed (in the forward of 1913 to the 2nd edition of LU) that the Prolegomena is, in its essential content, a mere reworking of two complementary series of lectures given in Halle in the summer and autumn of 1896. (LU, B XII; LU, 47)
Thus, as Holenstein writes,

it is remarkable that Husserl's own commentary on the "Psychological Studies" within his "Bericht über deutsche Schriften zur Logik aus dem Jahre 1894" written in Winter 1896/97, and published in 1897, i.e. after Frege's review...and after his 1896 lectures on the psychologism problem, contains not the slightest qualification of his earlier exposition of the "psychology of cognition and logic", indeed in this review Husserl still demands "the descriptive and genetic exploration" of the phenomena of intuition, representation and, in some circumstances, apperception for the "foundation of every theory of judgment".

(loc.cit.)

Thus certainly there is no sudden turn on Husserl's part which could be ascribed to some particular influence such as Frege's review of PdA, but it is still possible to award a place to Frege as someone who motivated some of the thought-experiments which Husserl must be presumed to have made throughout this period - a similar position can also, as remarked, be awarded to Natorp. Two further sets of factors seem to be of equal importance however: first, the technical problems which Husserl encountered in trying to extend the methods of PdA (which, we must remember, was only the first volume of a projected two-volume work) to cover, in the projected second volume, not merely arithmetic but all deductive theories with their correlate manifolds or domains of theoretical exploration. For it must be remembered that the analyses of PdA (as published) were confined largely to acts of counting and collecting together, of manipulating and understanding symbolism, and of abstraction erected on the basis of these lower-order acts. These are all areas where the psychological-clarificatory method is hardly stretched in ways which would reveal its fundamental inadequacy to the problems in hand (as Wittgenstein's behavioural approach was hardly stretched by the similar problems which he considered in his 1956). The second set of factors arose from Husserl's coming to appreciate the work of Bolzano and Lotze (see Husserl,1903 and Willard,1972, pp.98-99) and in particular to appreciate the logical platonism which these thinkers in their different ways advanced. The analysis of both these sets of factors will not escape our attention in the chapters which follow.