Introduction

Objective
To formulate an energy-stable high-order scheme for solving challenging fluid flow problems

Motivation
High-order flow solvers:
- Provide cost-effective high-fidelity solutions to real-world problems in fluid dynamics
- Produce less numerical dissipation than their low-order counterparts
- Resolve complex features in vortex-dominated flows, e.g., flows over
  - flapping wings
  - rotorcraft blades
  - high-lift configurations
- Can be difficult to implement and are more susceptible to numerical instabilities

Numerical Results

Theoretical Results

Recent developments
- ESFR schemes extended from 1-D to higher dimensions using tensor product formulations in
  - 2-D on quadrilateral elements
  - 3-D on hexahedral elements
- Stability of ESFR schemes proven for linear advection-diffusion problems (2nd order PDEs) in
  - 1-D on linear elements
  - 2-D on triangular elements
- Stability proven by
  - re-writing 2nd order systems as 1st order systems
  - introducing auxiliary variable \( \phi \) in place of each first derivative of the solution
  - obtaining upper bound on Sobolev-type norm of solution and auxiliary variable

Conclusions and Future Work

Summary
- Developed a class of provably stable high-order methods for 1st and 2nd order PDEs
- Extended formulation to multiple dimensions and unstructured grids of mixed elements

Future work will involve applications of the schemes to flows around deforming geometries and high-Re flows

Framework

Flux Reconstruction (FR)
- Unifies a number of well-known high-order methods, including certain Discontinuous Galerkin (DG) and Spectral Difference (SD) methods
- Simplifies implementation of high-order schemes (its formulation does not require explicit numerical quadratures)
- Gives rise to a new class of schemes with favorable accuracy and stability properties:
  - Energy-Stable Flux Reconstruction (ESFR) schemes

Procedure Description

ESFR Procedure for 1-D scalar conservation law (1st order PDE):

1. Solution \( u \) is approximated by \( \bar{u} \), a polynomial of degree \( p \)

2. Flux \( f \) is approximated by \( f^D \), a piecewise polynomial of degree \( p \)

3. A numerical flux is computed at the interface between elements

4. \( f^D \) is extrapolated to the left and right interfaces, yielding \( f^L \) and \( f^R \)

5. Degree \( p + 1 \) correction functions \( h_k \) and \( h_k^* \) are defined

6. Continuous flux is constructed:

   \[
   f = f^D + (f^L - f^D)h_k + (f^R - f^D)h_k^*
   \]

Recent developments
- Certain ESFR schemes have been shown to possess CFL limits which are 2x those of the nodal DG scheme
- Schemes have been successfully applied to linear advection-diffusion problems and the non-linear Navier-Stokes equations on mixed grids
- Results of some representative numerical experiments appear in Figures 1–5