High Order Spectral Difference Method for the Navier-Stokes Equations on Moving Deformable Meshes

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Introduction

The commonly used Computational Fluid Dynamic methods tend to be of second order accuracy. Current CFD research has seen high order methods emerging as promising and competitive tools for solving both steady and unsteady fluid dynamic problems. Extensions of high-order methods to dynamic deformable meshes and fluid structural interaction problems are areas of ongoing research. The ability to handle mesh deformation and fluid structure dynamics allows us to expand the application of CFD into a wider field of engineering. This study aims to demonstrate both capabilities using a recently developed high-order accurate CFD method called Spectral Difference method.

Numerical Method: Spectral Difference

Spectral Difference (SD) method is a Discontinuous Galerkin (DG) like finite element method. It is formulated using the differential form of the Navier-Stokes equation (hence its name), in contrast to the integral form of DG. The formulation is simpler and hence more computational efficient than DG method. The SD method, like DG, is element-wise discontinuous. Within each element, the solution and flux points are used to store flow solution and flow information. The solution points in each dimension are chosen to be the Chebyshev points. The flux points are selected to be the zeros of the Legendre polynomial with its end points. This choice leads to a stable SD scheme.

Solution and Flux Collocation Points

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Moving Deforming Mesh Technique

The grid deformation strategy implemented in the current study is an algebraic method that updates the mesh at every time step. This method has the property of preserving grid orthogonality near the surface under substantial deformation, which is very desirable for high Reynolds number viscous flow simulation. For unsteady moving boundary problems, the computational mesh needs to move with the boundary. When applying the spectral difference method to those kind of problems, it necessitates a way to handle the mesh motion without compromising the method’s accuracy. An effective way to achieve this is to implement a time dependent domain transformation between the fixed computational domain, where the spectral difference method is discretized and solved, and the moving physical domain where boundaries are displaced.

Accuracy Study on Deforming Mesh with Euler Vortex

The numerical discretization is performed in the stationary reference domain. The unsteady physical space solution is obtained through inverse mapping.

Flow Wake Interaction with Elastic Beam Structure

The deforming mesh framework can be extended to handle fluid structure interaction problem. This is currently work in progress. Numerical simulation for flow over a cylinder with an elastic beam attached at the rear end has been performed with Mach=0.1 and Re=200. The vorticity contours at various time instances are shown below. The beam motion is prescribed. Finite element solver will be added in the upcoming work.

Conclusions

Numerical simulation of flow over a plunging naca12 airfoil at Re=1,850 produces wake pattern that closely resembles the corresponding experimental result.

Further Information

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Bibliography


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