MICROECONOMICS
Comprehensive Examination
July 2018

Instructions:

1. Please answer each of the four questions on separate pieces of paper.

2. Please write only on one side of a sheet of paper.

3. Please write in pen only.

4. When finished, please arrange your answers in the order in which they appeared in the questions, i.e. 1(a), 1(b), etc.
Question 1. Define an inspection game as follows: there are three expected-utility maximizing players, a principal (P) and two agents (1 and 2). Players choose actions simultaneously. The principal can inspect either agent 1 ($a_P = 1$) or agent 2 ($a_P = 2$), and each agent $i$ can either work ($a_i = 1$) or shirk ($a_i = 0$).

The principal’s payoff depends only on the actions of the agents: she gets a payoff of 2 if both agents work, a payoff of 1 if one agent works and the other shirks, and a payoff of 0 if both shirk. An agent gets a payoff of 0 if he works (regardless of whether or not the principal inspects him), a payoff of 1 if he shirks and the principal does not inspect him, and a payoff of $-c < 0$ if he shirks and the principal inspects him.

(a) Draw the strategic/normal form of this game.

(b) First suppose that $c > 1$. Find a Nash equilibrium where the principal gets an expected payoff of 2.

(c) Continue to suppose that $c > 1$. Show that in any Nash equilibrium, the principal’s expected payoff is at least 1.

(d) Now suppose that $c < 1$. Find a Nash equilibrium where the principal gets an expected payoff of 0.

(e) Continue to suppose that $c < 1$. Show that in any Nash equilibrium, the principal’s expected payoff is at most 1.
Question 2. Let $\pi(p)$ denote the profit function of a firm whose production set is $Y \in \mathbb{R}^n$ and who faces prices $p \in \mathbb{R}_{++}^n$. The production set $Y$ is closed and satisfies the free disposal property. Assume that the production vector $y(p) = (y_1(p), \ldots, y_n(p))$ is the unique solution to the firm’s profit-maximization problem for each $p \in \mathbb{R}_{++}^n$, and let $\pi(p)$ denote the maximized profit. Throughout this problem assume as much differentiability as needed.

(a) Prove Hotelling’s Lemma ($\frac{\partial \pi(p)}{\partial y_i} = y_i(p)$).

(b) Argue that $Dy(p)$, the $n \times n$ matrix of partial derivatives whose $ij_{th}$ element is

$$\frac{\partial y_i(p)}{\partial p_j},$$

is positive semidefinite. What does this say about the slope of the firm’s output supply functions and input demand functions as functions of own prices?

(c) Suppose that you are given a function $\bar{y}(p) \in \mathbb{R}^n$ defined for $p \in \mathbb{R}_{++}^n$. Describe minimal conditions on $\bar{y}(p)$ and its partial derivatives $D\bar{y}(p)$ which would guarantee that $\bar{y}(p)$ is a supply function of some competitive firm. Explain your reasoning. (An outline of the main arguments here suffices.)
Question 3. Suppose that the continuous utility function $U : \mathbb{R}^n_+ \to \mathbb{R}$ represents continuous and locally non-satiated preferences and that $e(p, u)$ is its associated expenditure function. For each $x \in \mathbb{R}^n_+$, define

$$V(x) \equiv \max\{u : p \cdot x \geq e(p, u) \text{ for all } p \gg 0\}.$$

(a) Show that $V(\cdot)$ is increasing on $\mathbb{R}^n_+$.

(b) Show that $V(\cdot)$ is quasiconcave on $\mathbb{R}^n_+$.

(c) Show that $V(x) \geq U(x)$ for all $x \in \mathbb{R}^n_+$. Find an example of a utility function where $V(x) > U(x)$ for some $x \in \mathbb{R}^n_+$. A graph of the function or its indifference curves here will suffice.

(d) Consider the following two maximization problems, where $p \gg 0$:

$$U^*(p, I) \equiv \max\limits_{x \in \mathbb{R}^n_+} U(x) \text{ s.t. } p \cdot x \leq I$$

and

$$V^*(p, I) \equiv \max\limits_{x \in \mathbb{R}^n_+} V(x) \text{ s.t. } p \cdot x \leq I.$$  \hspace{1cm} (1)

Show that $U^*(p, I) = V^*(p, I)$. (Note that part (c) implies that $U^*(p, I) \leq V^*(p, I)$, so it remains only to show that $U^*(p, I) \geq V^*(p, I)$.) Argue that if $x^*$ is a solution to problem (i), then it is also a solution to (ii).

(e) Based on the analysis above, is it possible to distinguish between locally non-satiated preferences and increasing preferences based upon demand data alone? Is it possible to distinguish between non-convex and convex preferences based upon demand data alone? (HINT: it may be helpful to draw diagrams for the case of two goods to support your answers.)
**Question 4.** There is a single firm, Firm 1, that has a license to operate in a particular market. The total profit in the market, \( \pi \), is known to Firm 1 but not to anyone else. Everyone else knows only that the value of \( \pi \) was drawn from a uniform distribution on \([0, 2] \).

The government plans to auction a second license to operate in the market. The two bidders are Firm 1 and Firm 2. If Firm 2 wins the second license, then the two firms will share the market: each will earn profit \( \pi/2 \) (minus any auction payments). If Firm 1 wins the second license, OR if neither firm wins the second license, then Firm 1 earns profit \( \pi \) and Firm 2 earns 0 (minus any auction payments).

This structure is common knowledge. All agents are expected-profit maximizers.

4.1 First suppose that the government sells the second license using a second-price, sealed-bid auction with reserve price \( r \geq 0 \).

(a) What is the distribution of \( \tilde{\pi} \)? What is the set of (pure) bidding strategies for Firm 1? For Firm 2? What is the set of mixed strategies for Firm 2?

(b) Suppose that \( r = 0 \). Find an equilibrium pair of bidding strategies \( (b_1^*, b_2^*) \) with the property that Firm 2 randomizes uniformly over all bids between 0 and 1, or show that no such equilibrium exists.

(c) Suppose that \( r > \frac{1}{2} \). Is there an equilibrium where neither firm ever submits a bid? Explain.

(d) Suppose that \( r \in (0, \frac{1}{2}) \). Is there an equilibrium where neither firm ever submits a bid? Explain.

4.2 Now suppose that the government sells the second license using a first-price, sealed-bid auction with reserve price \( r = 0 \).

(e) Find an equilibrium pair of bidding strategies \( (b_1^{**}, b_2^{**}) \) with the property that Firm 2 randomizes uniformly over all bids between 0 and \( \frac{1}{2} \), or show that no such equilibrium exists.