MICROECONOMICS
Comprehensive Examination
June 2018

Instructions:

1. Please answer each of the four questions on separate pieces of paper.

2. Please write only on one side of a sheet of paper.

3. Please write in pen only.

4. When finished, please arrange your answers in the order in which they appeared in the questions, i.e. 1(a), 1(b), etc.
Question 1. A principal has an opportunity to hire an agent to undertake a project. The agent’s skill level $\theta$ is distributed uniformly on the interval $[0,1]$. The agent observes his skill level, but the principal does not. Both principal and agent are expected-utility maximizers.

The principal’s utility is quasilinear in money. The project yields a payoff of 1 to the principal if it succeeds, and 0 if it fails. The probability of success is equal to the agent’s skill level.

The principal’s outside option if she does not hire the agent is 0. Thus, her utility is $1 - x$ if she pays the agent $x$ and the project succeeds; $-x$ if she pays the agent $x$ and the project fails; and 0 if she does not hire the agent.

The agent’s utility depends only on his income: his utility from income $x$ is $\sqrt[3]{x}$. His outside option is an income $w_0$ that depends on his type: $w_0(\theta) = \frac{1}{2}\theta$. (That is, his outside option gives him utility $\sqrt[3]{\frac{1}{2}\theta}$.)

a) Suppose that the principal offers a “fixed-wage” contract that gives the agent a payment of $w \geq 0$, regardless of whether or not the project succeeds. What is the principal’s expected utility, as a function of $w$? (HINT: Which types of agent will accept the contract?)

b) What is the set of optimal fixed-wage contracts for the principal?

c) Now suppose that instead the principal offers a “bonus” contract that gives the agent a payment of $b \geq 0$ if the project succeeds and 0 otherwise. What is the principal’s expected utility, as a function of $b$? (HINT: Which types of agent will accept the contract?)

d) What is the set of optimal bonus contracts for the principal?
Question 2.

a) Consider the game tree in Figure 1, where the value of $x$ is 10 with probability $\epsilon \in (0, \frac{1}{2})$ and 0 with probability $1 - \epsilon$. Player 1 observes the value of $x$ but Player 2 does not. This structure is common knowledge. Find the unique perfect Bayesian equilibrium of this game.

b) Consider a sealed-bid, first-price, no-reserve auction with two bidders. The valuations $v_1$ and $v_2$ are independent random variables: $v_1$ is distributed uniformly on $[0, 1]$, and $v_2$ is distributed uniformly on $[v, \overline{v}]$, where $0 < v < 1 < \overline{v}$. Show that in any Bayes-Nash equilibrium in which each bidder’s bid function $b^*_i(v_i)$ is continuous and strictly increasing in $v_i$, it must be the case that $b^*_1(1) = b^*_2(\overline{v})$.

c) Two objects, $A$ and $B$, are auctioned off to two bidders, $i \in \{1, 2\}$. The value of good $X \in \{A, B\}$ to bidder $i$ is $v^X_i$, and if bidder $i$ obtains both goods, then her value is $v^A_i + v^B_i$. (That is, values are additive.) The values $v^X_i$ are independent and identically distributed on support $[0, 1]$ according to a smooth density $f(\cdot)$.

Show that the expected revenue generated by two separate ascending price auctions (one for each object) is lower than the expected revenue generated by a bundle ascending price auction in which the bundle $\{A, B\}$ is auctioned off as a whole.
**Question 3.** Consider an exchange economy with two goods and two types of consumers. Both types of consumers have $u(x, y) = \min\{x, y\}$ as their utility function. The endowment of a type one consumer is $e_1 = (2, 0)$, while that of a type two consumer is $e_2 = (0, 2)$. **Both goods are indivisible beyond one unit.** The economy is now replicated once so there are two consumers of each type.

We will consider two alternative definitions of what it means for a coalition to “block” an allocation:

3.1 First, say that a coalition blocks a given allocation if it can propose an alternative allocation (using only their own endowments) that makes *every* member of the coalition weakly better-off and at least one member strictly better-off.

(a) Under this definition, is every core allocation efficient?

(b) Under this definition, does every core allocation have the equal treatment property?

3.2 Second, now say that a coalition blocks a given allocation if it can propose an alternative allocation (using only their own endowments) that makes *every* member of the coalition strictly better-off.

(c) Find as many core allocations as you can in this case, and argue why each one of them is in the core.

(d) From (c) what can you say about the efficiency and equal treatment properties of core allocations in this case?
Question 4.

(a) Define competitive equilibrium for an economy with production. Use the following notation:

\[(u^i, e^i, \theta^{ij}, Y^j)_{i \in I, j \in J}.\]

(b) In the definition of a competitive equilibrium, why must we insist that the equilibrium price of a good be zero when it is in excess supply? I.e., what would be logically inconsistent with the model if we did not require this?

(c) Now consider a pure exchange economy with two consumers and two commodities. Consumer 1’s utility function is

\[u_1(x, y) = \max\{x, y\}\]

and consumer 2’s preferences are lexicographic, i.e.,

\[(x, y) \succeq_2 (x', y')\]

iff \(x > x'\) or \([x = x'\) and \(y \geq y']\). If each consumer has an endowment in \(\mathbb{R}^2_+\), then show that there is no competitive equilibrium for this economy in which the equilibrium price \(p\) is in \(\mathbb{R}^2_+ \setminus \{0\}\). Which of the assumptions of the existence theorem fail here?