1 First Semester

1.1 Alternative Tax Policies (45 Points)

Households have time-separable utility over consumption and leisure with a period-utility function of the form $u(c) + \nu(\ell)$, where both $u$ and $\nu$ are smooth, strictly increasing and strictly concave functions. The discount factor is $\beta$. The total time endowment is normalized to one for each household. The production technology is given by $F(k, n)$ and exhibits constant returns to scale.

1. Define a Tax-Distorted Competitive Equilibrium in the Neo-Classical growth model with taxes on labor income (tax rate $\tau_{n,t}$), gross capital income net of depreciation (i.e., $(r_t + 1 - \delta)k_t$ is taxed at rate $\tau_{k,t}$), and consumption expenditures (taxed at rate $\tau_{c,t}$). Prove that there is a unique non-trivial steady state if all tax rates are constant in $(0, 1)$.

2. Consider a steady state in which the consumption tax rate is zero and the capital tax rate is positive. Show that there is an alternative tax system in which the capital tax rate is zero but there are positive consumption expenditure taxes. What must be true about the sequence of tax rates $(\tau_{c,t})_{t=0}^{\infty}$ in order for the two economies to have the same allocations?

3. Consider the problem of a Ramsey planner choosing optimal taxes rates. Assume that the optimal tax system is interior (i.e. the first-order conditions are satisfied). Is it true that consumption taxes must converge to zero in order for the TDCE to reach the optimum? Explain.
1.2 OLG in With Different Trading Opportunities (45 Points)

Consider an small open economy with an overlapping generations structure. That is, every period \( t \) there are two types of households in the economy, those who have just been born and those that were born in \( t - 1 \). Endowments are given by \( e_y \) when a household is young and \( e_o \) when they are old. Households value consumption only in the two periods in which they live according to:

\[
U^t = \frac{1}{1 - \sigma} \left[ (c^t_l)^{1-\sigma} + (c^t_{l+1})^{1-\sigma} \right]
\]

(a) Suppose the economy is closed, i.e. the only trade that can occur is between households of consecutive generations. Define and fully characterize a competitive equilibrium under the assumption of no free disposal.

(b) Now suppose that for \( t \geq 0 \) the households can borrow or save from abroad at the exogenous world interest rate \( 1 + r \). The initial old (those born at \( t = -1 \)) have no debt or savings. Change the definition of equilibrium (i.e. add or remove one equation) to allow for this borrowing and saving from abroad and fully characterize the equilibrium as a function of \( r \).

(c) Suppose now that there are two countries that can trade with one another. They have the same number of young and old households in each period. Country one has endowments \( e^1_y = 1 + \epsilon \) and \( e^1_o = 1 - \epsilon \). Country two has endowments \( e^2_y = 1 - \epsilon \) and \( e^2_o = 1 + \epsilon \). Define a world-wide equilibrium when the countries can borrow and lend between one another (but not from outside of these two). What is the market clearing interest rate? Hint - guess and verify after taking note of the symmetry!

(d) Now suppose that country one has fiat money. The initial old households in country one own the entire stock, given by \( M = \epsilon \), at \( t = 0 \) and there are no money injections over time. Is there a monetary equilibrium in which country one has valued fiat money? If so then what is true about global welfare compared to the equilibrium in part (d)?
2 Second Semester

2.1 Skilled and unskilled labor in a RBC model (45 points)

Consider the following closed-economy RBC model. There is no growth for simplicity.

Preferences
A representative household supplies two distinct types of labor, skilled and unskilled, and its expected discounted utility is given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t^s, L_t^u) \]

where \( E_0 \) is the conditional expectation operator, \( C_t \) is consumption, \( L_t^s \) is skilled labor hours, \( L_t^u \) is unskilled labor hours, and \( 0 < \beta < 1 \) is the discount factor. The period utility \( u(\cdot) \) satisfies standard properties.

Production Technology
In this economy, output \( (Y_t) \) is produced using a production function

\[ Y_t = A_t F(K_t, N_t^s, N_t^u) \]

where \( K_t \) is (pre-determined) capital, \( N_t^s \) is skilled labor hours, and \( N_t^u \) is unskilled labor hours. The production function \( F(\cdot) \) is homogenous of degree one and satisfies other standard properties. \( A_t \) is a random productivity shock that follows a stationary process. (Note that the two types of labor are not perfect substitutes in the production function.)

Accumulation Technology
The evolution of capital \( K_t \) is given by

\[ K_{t+1} = I_t + (1 - \delta) K_t \]

where \( I_t \) is investment and \( 0 < \delta < 1 \) is the rate of depreciation.

Market Clearing
The market clearing conditions for the two types of labor are given by

\[ L_t^s = N_t^s, \quad L_t^u = N_t^u. \]

Moreover, since total output produced can be either consumed by the household or invested, another resource constraint is

\[ Y_t = C_t + I_t. \]

(i) Formulate the social planner’s version of the above model.
(ii) Formulate a price-taking version of the above model in which the representative household owns the capital stock and makes investment decision. Define carefully the competitive equilibrium based on your formulation.
(iii) Do the allocations from the planner’s problem in (i) and the competitive equilibrium in (ii) coincide? Defend your answer fully.
(iv) Consider the following functional forms for preferences and technology

\[
\begin{align*}
    u(C_t, L_s^t, L_u^t) &\equiv \log C_t - \chi \frac{(L_s^t)^{1+\phi}}{1+\phi} - \chi \frac{(L_u^t)^{1+\phi}}{1+\phi} \\
    F(K_t, N_s^t, N_u^t) &\equiv \left[ \mu (N_u^t)^\sigma + (1-\mu) \left[ \lambda (K_t)^\rho + (1-\lambda) (N_s^t)^\rho \right] \right]^{\frac{1}{\sigma}}
\end{align*}
\]

where $\chi, \phi > 0$, $0 < \mu < 1$, $0 < \lambda < 1$ and $\sigma, \rho > 1$.

Express equilibrium relative wages, that is, ratio of the two types of wages, as a function of relative supply of the two types of labor, and the capital to skilled labor ratio.

(v) Show that the ratio of skilled wage to unskilled wage is increasing in the capital to skilled labor ratio if

\[
\sigma > \rho.
\]

Interpret this result fully.
2.2 Optimal monetary and fiscal policy (45 points)

The government’s objective is to minimize the loss function

\[
\frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left[ \phi_\pi \pi^2_{t+j} + \phi_x x^2_{t+j} + \phi_s s^2_{t+j} \right]
\]

subject to

\[
\begin{align*}
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + \varepsilon_{\pi,t} \\
x_t &= E_t x_{t+1} - (i_t - E_t \pi_{t+1}) + \varepsilon_{x,t} \\
m_t &= \eta_x x_t - \eta_i i_t + \varepsilon_{m,t} \\
s_t &= m_t - m_{t-1} + \pi_t \\
b_t &= \beta^{-1} b_{t-1} - \beta^{-1} \pi_t + i_t - \beta^{-1} s_t
\end{align*}
\]

where \( E_t \) is the conditional expectation operator, \( m_t \) is the policy instrument, \( \pi_t, x_t, s_t, b_t, \) and \( i_t \) are other endogenous model variables, and \( 0 < \beta < 1, \kappa > 0, \eta_x > 0, \eta_i > 0, \phi_\pi > 0, \phi_x > 0, \phi_s > 0 \) are model parameters. The government takes actions after the shocks \( \varepsilon_{\pi,t}, \varepsilon_{x,t}, \) and \( \varepsilon_{m,t} \) are realized. The shocks \( \varepsilon_{\pi,t}, \varepsilon_{x,t}, \) and \( \varepsilon_{m,t} \) are iid over time and have unit variance.

(For model based interpretation, \( \pi_t \) is inflation, \( y_t \) is output, \( i_t \) is the one-period nominal interest rate, \( b_t \) is the real value of one-period nominal government debt, \( m_t \) is real balances, and \( s_t \) is seigniorage.)

(i) First, suppose that the government can credibly commit at date \( t \) to a contingent path for \( m_{t+j} \). Characterize, as far as you can, the solution to the optimal policy problem above with commitment.

(ii) Does the solution above in (i) feature dynamic time-inconsistency? Defend your answer.

(iii) Next, suppose that the government cannot credibly commit and, instead, chooses \( m_t \) at each date. Characterize, as far as you can, the (Markov-perfect) solution to the optimal policy problem above without commitment.