Macroeconomic Theory  
Spring 2016

Comprehensive Exam

There are 4 questions and a total of 180 points.
1. Government spending in a RBC model (35 points)
Consider the following closed-economy RBC model. There is no growth for simplicity.
Preferences
The representative household’s expected discounted utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$$

where $E_0$ is the conditional expectation operator, $C_t$ is consumption, $L_t$ is leisure, and $0 < \beta < 1$ is
the discount factor. The period utility $u(\cdot)$ is strictly increasing, concave, and twice continuously
differentiable.

Production Technology
In this economy, output ($Y_t$) is produced using a production function

$$Y_t = A_t F(K_t, N_t)$$

where $K_t$ is (pre-determined) capital, $N_t$ is labor, and $A_t$ is a random productivity shock. The
production function $F(\cdot)$ is twice continuously differentiable, concave, and homogenous of degree
one. $F(\cdot)$ also satisfies the standard limiting conditions (the Inada conditions).

Accumulation Technology
The evolution of capital is given by

$$K_{t+1} = I_t + (1 - \delta) K_t$$

where $I_t$ is investment and $0 < \delta < 1$ is the rate of depreciation.

Government
The government consumes an exogenous and random amount $G_t$ every period. The government
finances its consumption using lump-sum taxes.

Resource Constraints
The total amount of time that the household has can be split into work and leisure. Normalizing
the total amount of time each period to be 1, the time constraint is

$$N_t + L_t = 1.$$ 

Moreover, since total output produced can be either consumed by the household and government
or invested, another resource constraint is

$$Y_t = C_t + I_t + G_t.$$ 

(i) (10 points) Formulate a price-taking version of the above model in which the representative
firm owns the capital stock and issues equity to finance investment.

(ii) (5 points) Define the competitive equilibrium based on your formulation in (i) above.
(iii) (5 points) Derive all the conditions (optimality and market clearing) that characterize the competitive equilibrium.

Now suppose that government spending enters household utility. The household’s period utility $u(\cdot)$ is then given by

$$u(C_t, L_t, G_t).$$  \hfill (1)

(iv) (5 points) Depending on the properties of the period utility given in (1), argue carefully if the competitive equilibrium (without necessarily solving for it fully) will lead to different allocations than those above in (iii).

(v) (10 points) Given the period utility in (1), consider now the government optimally determining the level of government spending. What principle/optimality condition will determine the level of government spending in this case?

(You do not have to fully solve the model for equilibrium under optimal government spending policy. Note also that the rest of the model, including production and technology technologies and resource constraints, remain the same as in (i)-(iii).)
2. Optimal monetary policy in a sticky price model (55 points)

The central bank’s objective is to minimize the loss function

\[ \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left[ \phi_\pi \pi_t^2 + \phi_x x_t^2 + \phi_i i_t^2 \right] \]

subject to

\[ \left( \pi_t - \alpha \pi_{t-1} \right) = \beta E_t \left[ \pi_{t+1} - \alpha \pi_t \right] + \pi_t \left( x_t - \lambda x_{t-1} \right) + \epsilon_t \]
\[ \left( x_t - \lambda x_{t-1} \right) = E_t \left[ x_{t+1} - \lambda x_t \right] - \left( i_t - E_t \pi_{t+1} \right) \]

where \( E_t \) is the conditional expectation operator, \( i_t \) is the central bank’s instrument, \( \pi_t \) and \( x_t \) are other endogenous model variables, and \( 0 < \beta < 1, 0 < \alpha < 1, 0 < \lambda < 1, \kappa > 0, \phi_\pi > 0, \phi_x > 0, \phi_i > 0 \) are model parameters. The central bank takes actions after the shock \( \epsilon_t \) is realized. \( \epsilon_t \) is iid over time and has unit variance.

(i) (20 points) First, suppose that the central bank can credibly commit at date \( t \) to a contingent path for \( i_{t+j} \). Characterize, as far as you can, the solution to the optimal monetary policy problem above with commitment. Does the solution feature dynamic time-inconsistency? Defend your answer.

(ii) (35 points) Next, suppose that the central bank cannot credibly commit and, instead, chooses \( i_t \) at each date. Characterize, as far as you can, the (Markov-perfect) solution to the optimal monetary policy problem above without commitment.
3. **Over-Lapping Generations (45 pts)** Consider a Lucas Tree economy with three overlapping generations. That is, for any period $t \geq 0$ there are three households alive, the young, middle-aged, and old. Preferences are given as follows:

$$U^t(c^t_1, c^t_{t+1}, c^t_{t+2}) = \sum_{\ell=0}^{2} \psi_{\ell} \frac{(c^t_{t+\ell})^{1-\sigma}}{1-\sigma}$$

Where $\psi_0 = 1$ and $\psi_1, \psi_2$ are general utility weights.

Households receive endowments only when they are alive, the profile of which is constant: $e^t = (e_y, e_m, e_o)$ with $e_y + e_m + e_o = 1 - \theta$. In addition, there is a Lucas Tree in the economy, which produces $\theta$ units of consumption good in each period. There is an initial old household in $t = 0$ who is endowed with $s_o$ shares of the tree (in addition to $e_o$ units of good) and an initial middle-aged who is endowed with $s_m = 1 - s_o$ units of the tree (in addition to $e_m$ units of good). Young households are always born with zero shares of the tree.

1. Define a Sequential Markets Equilibrium for this economy.
2. Show that the equilibrium of this economy can be characterized by a system of difference equations in $s_{m,t}$ and $p_t$ (the price of the tree).
3. Is it always Pareto Optimal to have consumption constant across age groups? If not, then give a condition for it to be so.
4. Maintain the condition from (3). Can you necessarily find a sequence of prices so that the constant consumption allocation is consistent with equilibrium? If not then give a condition so that the constant consumption allocation is an equilibrium.
4. Optimal Fiscal Policy (45 pts) Consider an economy with a pollution externality. That is, for every unit of output there is some amount of pollution generated, call it $\Phi_t = \phi Y_t$. The production technology is neo-classical with capital and labor as inputs and a constant Solow Residual. Capital is accumulated according to the standard law of motion, and households have life-time utility given by:

$$U = \sum_{t=0}^{\infty} \beta^t \left[ u(c_t, \ell_t) - \Phi_t \right]$$

Where $\ell_t \in [0, 1]$ is leisure. The government must finance an exogenous sequence of government expenditures, $(g_t)_{t=0}^{\infty}$.

1. Characterize the set of Pareto Efficient allocations in this economy.

2. Define and characterize the Tax-Distorted Competitive Equilibrium in this economy when proportional taxes are levied on labor and capital income. Households should take all prices, taxes, and aggregate variables as exogenous.

3. Derive the Implementability Condition for this economy and set up the Ramsey Planner’s problem.

4. What happens to the capital income tax rate as $t \to \infty$?

5. Suppose that the Ramsey Planner had access to lump-sum taxes. Would the Ramsey Planner use only lump sum taxes? Why or why not.